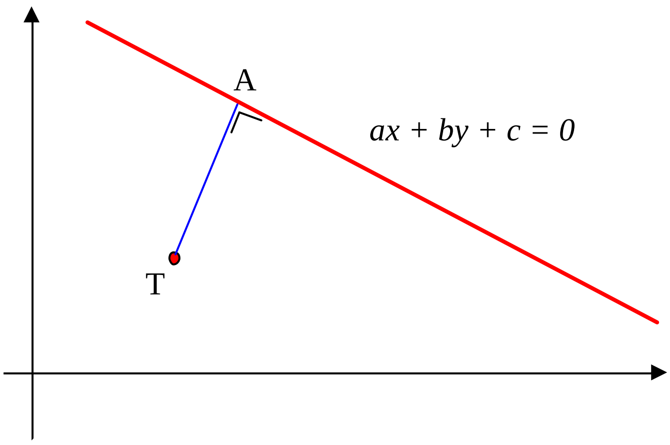
Rumus Jarak Titik dan Garis



Misalkan titik $T(x_1, y_1)$ dan $A(x_A, y_A)$

Jarak titik $T(x_1, y_1)$ ke titik $A(x_A, y_A)$ adalah

$d = \sqrt{(x_A - x_1)^2 + (y_A - y_1)^2}$ ....(1)

Titik $A(x_A, y_A)$ terletak pada garis $g \equiv ax + by + c = 0$, sehingga

$ax_A + by_A + c = 0 \;\Rightarrow\; ax_A + by_A = -c$

$\Rightarrow \quad x_A = \dfrac{-by_A - c}{a}$ atau $y_A = \dfrac{-ax_A - c}{b}$ ....(2)

Misalkan gradien garis g adalah m_g dan gradien garis AT adalah m_{AT} ,karena kedua garis tegak lurus, maka

$m_g \cdot m_{AT} = -1 \;\Rightarrow\; -\dfrac{a}{b} \cdot \dfrac{y_A - y_1}{x_A - x_1} = -1$

$\Rightarrow \quad \dfrac{-ay_A + ay_1}{bx_A - bx_1} = -1$

$\Rightarrow \quad bx_A - ay_A = bx_1 - ay_1$ .... (3)

Jika kita subtitusi persamaan (2) ke (3), akan kita peroleh

$$bx_A - a\left(\frac{-ax_A - c}{b}\right) = bx_1 - ay_1 \Rightarrow \frac{b^2 x_A}{b} - \frac{-a^2 x_A - ac}{b} = bx_1 - ay_1$$

$$\Rightarrow \quad b^2 x_A + a^2 x_A + ac = b^2 x_1 - aby_1$$

$$\Rightarrow \quad b^2 x_A + a^2 x_A = b^2 x_1 - aby_1 - ac$$

$$\Rightarrow \quad x_A = \frac{b^2 x_1 - aby_1 - ac}{a^2 + b^2}$$

demikian juga,

$$b\left(\frac{-by_A - c}{a}\right) - ay_A = bx_1 - ay_1 \Rightarrow \frac{-b^2 y_A - bc}{a} - \frac{a^2 y_A}{a} = bx_1 - ay_1$$

$$\Rightarrow \quad -b^2 y_A - bc - a^2 y_A = abx_1 - a^2 y_1$$

$$\Rightarrow \quad b^2 y_A + bc + a^2 y_A = -abx_1 + a^2 y_1$$

$$\Rightarrow \quad b^2 y_A + a^2 y_A = -abx_1 + a^2 y_1 - bc$$

$$\Rightarrow \quad y_A = \frac{a^2 y_1 - abx_1 - bc}{a^2 + b^2}$$

Subtitusi x_A dan y_A kepersamaan (1)

$$d=\sqrt{(x_A-x_1)^2+(y_A-y_1)^2}$$

$$=\sqrt{\left(\frac{b^2x_1-aby_1-ac}{a^2+b^2}-x_1\right)^2+\left(\frac{a^2y_1-aby_1-bc}{a^2+b^2}-y_1\right)^2}$$

$$=\sqrt{\left(\frac{b^2x_1-aby_1-ac}{a^2+b^2}-\frac{a^2x_1+b^2x_1}{a^2+b^2}\right)^2+\left(\frac{a^2y_1-abx_1-bc}{a^2+b^2}-\frac{a^2y_1+b^2y_1}{a^2+b^2}\right)^2}$$

$$=\sqrt{\left(\frac{b^2x_1-aby_1-ac-a^2x_1-b^2x_1}{a^2+b^2}\right)^2+\left(\frac{a^2y_1-abx_1-bc-a^2y_1-b^2y_1}{a^2+b^2}\right)^2}$$

$$=\sqrt{\left(\frac{-aby_1-ac-a^2x_1}{a^2+b^2}\right)^2+\left(\frac{-abx_1-bc-b^2y_1}{a^2+b^2}\right)^2}$$

$$=\sqrt{\left(\frac{-a(ax_1+by_1+c)}{a^2+b^2}\right)^2+\left(\frac{-b(ax_1+by_1+c)}{a^2+b^2}\right)^2}$$

$$=\sqrt{\frac{a^2(ax_1+by_1+c)^2+b^2(ax_1+by_1+c)^2}{(a^2+b^2)^2}}$$

$$=\sqrt{\frac{(a^2+b^2)(ax_1+by_1+c)^2}{(a^2+b^2)^2}}$$

$$=\sqrt{\frac{(ax_1+by_1+c)^2}{(a^2+b^2)}}$$

$$=\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$$

Karena jarak tidak pernah bernilai negatif, maka dapat disimpulkan bahwa:

Jarak titik $T(x_1,\ y_1)$ terhadap garis $ax+by+c=0$ adalah:

$$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$