

Rangkuman Materi dan Soal-soal



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Ringkasan Materi dan Contoh Soal



1. Pengertian

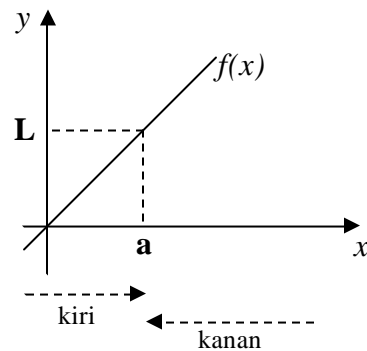
a). Limit kanan dan limit kiri

*) $\lim_{x \rightarrow a^+} f(x) = L$, artinya bilamana x mendekati a dari kanan, maka nilai $f(x)$ mendekati L .

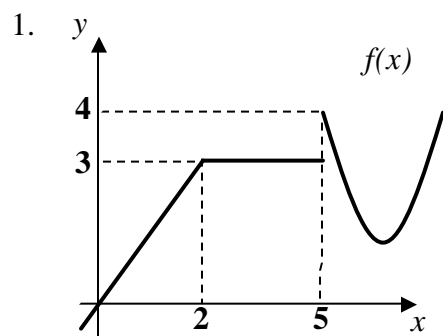
*) $\lim_{x \rightarrow a^-} f(x) = L$, artinya bilamana x mendekati a dari kiri, maka nilai $f(x)$ mendekati L .

b). Definisi limit

$$\lim_{x \rightarrow a} f(x) = L \text{ (ada)} \Leftrightarrow \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = L$$



Soal-soal:



Dari gambar diperoleh:

1). $\lim_{x \rightarrow 2^-} f(x) = 3$ dan $\lim_{x \rightarrow 2^+} f(x) = 3$ maka $\lim_{x \rightarrow 2} f(x) = 3$

2). $\lim_{x \rightarrow 5^-} f(x) = 3$ dan $\lim_{x \rightarrow 5^+} f(x) = 4$, limit kiri dan limit kanan tidak sama maka $\lim_{x \rightarrow 5} f(x) = \text{Tidak Ada}$

2. Jika $f(x) = \begin{cases} x+2; & jk x \leq -3 \\ 2-x; & jk x > 3 \end{cases}$ maka $\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^-} x+2 = -3+2 = -1$ dan

$\lim_{x \rightarrow -3^+} f(x) = \lim_{x \rightarrow -3^+} 2-x = 2-(-3) = 5$ sehingga $\lim_{x \rightarrow -3} f(x)$ tak ada (limit kiri \neq limit kanan)

3. Jika $f(x) = \begin{cases} 4x-1; & jk x < 2 \\ x^2+3; & jk x \geq 2 \end{cases}$ maka $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 4x-1 = 4 \cdot 2 - 1 = 8 - 1 = 7$ dan

$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x^2+3 = 2^2+3 = 4+3 = 7$ sehingga $\lim_{x \rightarrow 2} f(x) = 7$

2. Nilai Limit Fungsi Aljabar

Menentukan nilai limit $\lim_{x \rightarrow a} f(x)$ dengan cara:

a). **Substitusi**, jika diperoleh bentuk tak tentu $(\frac{0}{0})$, maka dilakukan:

b). Faktorisasi, atau

c). Perkalian dengan sekawan

❖ Untuk $\lim_{x \rightarrow a} f(x)$ dengan substitusi

➤ Jika $f(a) = c$ maka $\lim_{x \rightarrow a} f(x) = c$

➤ Jika $f(a) = \frac{c}{0}$ maka $\lim_{x \rightarrow a} f(x) = \infty$

➤ Jika $f(a) = \frac{0}{c}$ maka $\lim_{x \rightarrow a} f(x) = 0$

➤ Jika $f(a) = \frac{0}{0}$ maka dilakukan cara **b).** atau cara **c).**

Soal-soal:

1). $\lim_{x \rightarrow 3} (5x - 6) = 5 \cdot 3 - 6 = 15 - 6 = 9$

2). $\lim_{x \rightarrow -3} \frac{5x - 6}{x + 1} = \frac{5(-3) - 6}{-3 + 1} = \frac{-15 - 6}{-2} = \frac{-21}{-2} = \frac{21}{2}$

3). $\lim_{x \rightarrow 2} \frac{x - 2}{x + 2} = \frac{2 - 2}{2 + 2} = \frac{0}{4} = 0$

4). $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 5x + 6} = \frac{2 - 2}{2^2 - 5 \cdot 2 + 6} = \frac{0}{0}$ BTT, maka

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 5x + 6} = \lim_{x \rightarrow 2} \frac{x - 2}{(x - 2)(x - 3)} = \lim_{x \rightarrow 2} \frac{1}{(x - 3)} = \frac{1}{2 - 3} = \frac{1}{-1} = -1$$

5). $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^2 - 5x - 6} = \frac{(-1)^2 + 3(-1) + 2}{(-1)^2 - 5(-1) - 6} = \frac{1 - 3 + 2}{1 + 5 - 6} = \frac{0}{0}$ BTT, maka

$$\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^2 - 5x - 6} = \lim_{x \rightarrow -1} \frac{(x + 1)(x + 2)}{(x + 1)(x - 6)} = \lim_{x \rightarrow -1} \frac{(x + 2)}{(x - 6)} = \frac{-1 + 2}{-1 - 6} = \frac{1}{-7} = -\frac{1}{7}$$

6). $\lim_{x \rightarrow 0} \frac{x^3 - 5x^2 + 3x}{2x - 7x^2} = \frac{0^3 - 5 \cdot 0^2 + 3 \cdot 0}{2 \cdot 0 - 7 \cdot 0^2} = \frac{0}{0}$ BTT, maka

$$\lim_{x \rightarrow 0} \frac{x^3 - 5x^2 + 3x}{2x - 7x^2} = \lim_{x \rightarrow 0} \frac{x(x^2 - 5x + 3)}{x(2 - 7x)} = \lim_{x \rightarrow 0} \frac{(x^2 - 5x + 3)}{(2 - 7x)} = \frac{0 - 5 \cdot 0 + 3}{2 - 7 \cdot 0} = \frac{3}{2}$$

7). $\lim_{x \rightarrow 2} \frac{3 - \sqrt{4x + 1}}{x - 2} = \frac{3 - \sqrt{8 + 1}}{2 - 2} = \frac{0}{0}$ BTT, maka

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{3 - \sqrt{4x + 1}}{x - 2} &= \lim_{x \rightarrow 2} \frac{3 - \sqrt{4x + 1}}{x - 2} \cdot \frac{3 + \sqrt{4x + 1}}{3 + \sqrt{4x + 1}} = \lim_{x \rightarrow 2} \frac{9 - (4x + 1)}{(x - 2)(3 + \sqrt{4x + 1})} \\ &= \lim_{x \rightarrow 2} \frac{8 - 4x}{(x - 2)(3 + \sqrt{4x + 1})} = \lim_{x \rightarrow 2} \frac{-4(x - 2)}{(x - 2)(3 + \sqrt{4x + 1})} \\ &= \lim_{x \rightarrow 2} \frac{-4}{(3 + \sqrt{4x + 1})} = \frac{-4}{3 + \sqrt{4 \cdot 2 + 1}} = \frac{-4}{3 + 3} = -\frac{4}{6} = -\frac{2}{3} \end{aligned}$$

8). $\lim_{x \rightarrow 3} \frac{\sqrt{x + 2} - \sqrt{2x - 1}}{\sqrt{2x - 3} - \sqrt{x}} = \frac{0}{0}$ BTT, maka

$$\lim_{x \rightarrow 3} \frac{\sqrt{x + 2} - \sqrt{2x - 1}}{\sqrt{2x - 3} - \sqrt{x}} = \lim_{x \rightarrow 3} \frac{\sqrt{x + 2} - \sqrt{2x - 1}}{\sqrt{2x - 3} - \sqrt{x}} \cdot \frac{\sqrt{x + 2} + \sqrt{2x - 1}}{\sqrt{x + 2} + \sqrt{2x - 1}}$$

Dikali sekawan pembilang

$$= \lim_{x \rightarrow 3} \frac{(x + 2) - (2x - 1)}{(\sqrt{2x - 3} - \sqrt{x})(\sqrt{x + 2} + \sqrt{2x - 1})}$$

$$= \lim_{x \rightarrow 3} \frac{-x + 3}{(\sqrt{2x - 3} - \sqrt{x})(\sqrt{x + 2} + \sqrt{2x - 1})}$$

$$= \lim_{x \rightarrow 3} \frac{-x + 3}{(\sqrt{2x - 3} - \sqrt{x})(\sqrt{x + 2} + \sqrt{2x - 1})} \cdot \frac{(\sqrt{2x - 3} + \sqrt{x})}{(\sqrt{2x - 3} + \sqrt{x})}$$

Dikali sekawan penyebut

$$= \lim_{x \rightarrow 3} \frac{(-x + 3)(\sqrt{2x - 3} + \sqrt{x})}{(\sqrt{x + 2} + \sqrt{2x - 1})((2x - 3) - (x))}$$

$$= \lim_{x \rightarrow 3} \frac{-(x - 3)(\sqrt{2x - 3} + \sqrt{x})}{(\sqrt{x + 2} + \sqrt{2x - 1})(x - 3)}$$

$$= \lim_{x \rightarrow 3} \frac{-(\sqrt{2x - 3} + \sqrt{x})}{(\sqrt{x + 2} + \sqrt{2x - 1})}$$

$$= \frac{-(\sqrt{2 \cdot 3 - 3} + \sqrt{3})}{(\sqrt{3 + 2} + \sqrt{2 \cdot 3 - 1})} = \frac{-(\sqrt{3} + \sqrt{3})}{\sqrt{5} + \sqrt{5}} = -\frac{2\sqrt{3}}{2\sqrt{5}} = -\frac{\sqrt{3}}{\sqrt{5}}$$

Menentukan nilai limit $\lim_{x \rightarrow \infty} f(x)$ dengan cara:

a). **Substitusi.**

b). Jika diperoleh bentuk tak tentu ($\frac{\infty}{\infty}$) maka masing2 pembilang dan penyebut dibagi dengan variabel pangkat tertinggi (VPT).

c). Jika diperoleh bentuk tak tentu ($\infty - \infty$) maka dikalikan bentuk sekawannya kemudian masing2 pembilang dan penyebut dibagi dengan variabel pangkat tertinggi (VPT).

❖ Untuk $\lim_{x \rightarrow \infty} f(x)$ dengan substitusi

- Jika $f(x) = \frac{\infty}{c}$ maka $\lim_{x \rightarrow \infty} f(x) = \infty$
- Jika $f(x) = \frac{c}{\infty}$ maka $\lim_{x \rightarrow \infty} f(x) = 0$
- Jika $f(x) = \frac{\infty}{\infty}$ maka dilakukan dengan cara **b).**
- Jika $f(x) = \infty - \infty$ maka gunakan cara **c).**

Catatan:

- 1) $\lim_{x \rightarrow \infty} \frac{k}{x^n} = 0; n > 0$
- 2) $\lim_{x \rightarrow \infty} kx^n = \infty; n > 0$
- 3) $\lim_{x \rightarrow \infty} k = k; k$ konstanta

Soal-soal:

1). $\lim_{x \rightarrow \infty} 2x + 9 = 2 \cdot \infty + 9 = \infty$

2). $\lim_{x \rightarrow \infty} \frac{6}{x^2 + 1} = \frac{6}{\infty^2 + 1} = \frac{6}{\infty} = 0$

3). $\lim_{x \rightarrow \infty} 96 = 96$

4). $\lim_{x \rightarrow \infty} \frac{2x}{3x^2 + x - 1} = \frac{\infty}{\infty}$ BTT maka

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{2x}{3x^2 + x - 1} &= \lim_{x \rightarrow \infty} \frac{\frac{2x}{x^2}}{\frac{3x^2}{x^2} + \frac{x}{x^2} - \frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x}}{3 + \frac{1}{x} - \frac{1}{x^2}} = \frac{\lim_{x \rightarrow \infty} \frac{2}{x}}{\lim_{x \rightarrow \infty} 3 + \lim_{x \rightarrow \infty} \frac{1}{x} - \lim_{x \rightarrow \infty} \frac{1}{x^2}} \\ &= \frac{0}{3 + 0 - 0} = \frac{0}{3} = 0 \end{aligned}$$

Variabel Pangkat Tertinggi (VPT) adalah x^2 , maka pembilang dan penyebut dibagi dengan x^2

Lihat Teorema Limit

5). $\lim_{x \rightarrow \infty} \frac{2x^2}{3x^2 + x - 1} = \frac{\infty}{\infty}$ BTT, maka

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{2x^2}{3x^2 + x - 1} &= \lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^2}}{\frac{3x^2}{x^2} + \frac{x}{x^2} - \frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{2}{3 + \frac{1}{x} - \frac{1}{x^2}} = \frac{\lim_{x \rightarrow \infty} 2}{\lim_{x \rightarrow \infty} 3 + \lim_{x \rightarrow \infty} \frac{1}{x} - \lim_{x \rightarrow \infty} \frac{1}{x^2}} \\ &= \frac{2}{3 + 0 - 0} = \frac{2}{3} \end{aligned}$$

6). $\lim_{x \rightarrow \infty} (\sqrt{4x^2 - 5x + 1} - \sqrt{4x^2 + 7x - 2}) = \infty - \infty$ BTT, maka

$$\begin{aligned}
& \lim_{x \rightarrow \infty} (\sqrt{4x^2 - 5x + 1} - \sqrt{4x^2 + 7x - 2}) \\
&= \lim_{x \rightarrow \infty} (\sqrt{4x^2 - 5x + 1} - \sqrt{4x^2 + 7x - 2}) \cdot \frac{(\sqrt{4x^2 - 5x + 1} + \sqrt{4x^2 + 7x - 2})}{(\sqrt{4x^2 - 5x + 1} + \sqrt{4x^2 + 7x - 2})} \rightarrow \boxed{\text{Dikalikan sekawan}} \\
&= \lim_{x \rightarrow \infty} \frac{(4x^2 - 5x + 1) - (4x^2 + 7x - 2)}{\sqrt{4x^2 - 5x + 1} + \sqrt{4x^2 + 7x - 2}} \\
&= \lim_{x \rightarrow \infty} \frac{-12x + 3}{\sqrt{4x^2 - 5x + 1} + \sqrt{4x^2 + 7x - 2}} \longrightarrow \boxed{\text{Sama nilainya dengan (diambil suku yang memuat pangkat tertinggi dari pembilang dan penyebut):}} \\
&= \lim_{x \rightarrow \infty} \frac{-12x/x + 3/x}{\sqrt{4x^2/x^2 - 5x/x^2 + 1/x^2} + \sqrt{4x^2/x^2 + 7x/x^2 - 2/x^2}} \\
&= \lim_{x \rightarrow \infty} \frac{-12 + 3/x}{\sqrt{4 - 5/x + 1/x^2} + \sqrt{4 + 7/x - 2/x^2}} \longrightarrow \boxed{\text{VPT pembilang adalah } x, \text{ dan VPT penyebut } \sqrt{x^2} \text{ (setara), maka pembilang dan penyebut dibagi dengan } x \text{ (jkl dlm akar menjadi } x^2 \text{)}} \\
&= \frac{-12 + 0}{\sqrt{4 - 0 + 0} + \sqrt{4 + 0 - 0}} \\
&= -\frac{12}{2\sqrt{4}} = -\frac{12}{4} = -3 \quad \text{Lihat catatan 2}
\end{aligned}$$

Beberapa Kesimpulan untuk limit tak hingga:

➤ Jika $f(x) = \frac{ax^n + bx^{n-1} + \dots}{px^m + qx^{n-1} + \dots}$ maka $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{ax^n}{px^m} = \begin{cases} 0, & \text{jk } n < m \\ \frac{a}{p}, & \text{jk } n = m \\ \infty, & \text{jk } n > m \end{cases}$

n adalah pangkat tertinggi dari pembilang dan m adalah pangkat tertinggi dari penyebut.

➤ Jika $f(x) = \sqrt{ax^2 + bx + c} - \sqrt{px^2 + qx + r}$ maka $\lim_{x \rightarrow \infty} f(x) = \begin{cases} \infty, & \text{jk } a > p \\ \frac{b - q}{2\sqrt{a}}, & \text{jk } a = p \\ -\infty, & \text{jk } a < p \end{cases}$

3. Teorema Limit

Untuk $n \in$ bilangan bulat positif; c konstanta; f dan g fungsi-fungsi dalam x yang mempunyai limit di a , maka berlaku:

a. $\lim_{x \rightarrow a} c = c$	g. $\lim_{x \rightarrow a} (f(x) \cdot g(x)) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
b. $\lim_{x \rightarrow a} x^n = a^n$	h. $\lim_{x \rightarrow a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} ; \lim_{x \rightarrow a} g(x) \neq 0$
c. $\lim_{x \rightarrow a} f(x) = f(a)$	i. $\lim_{x \rightarrow a} (f(x))^n = (\lim_{x \rightarrow a} f(x))^n$
d. $\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)$	j. $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} ; \lim_{x \rightarrow a} f(x) \geq 0$
e. $\lim_{x \rightarrow a} (f(x) + g(x)) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$	
f. $\lim_{x \rightarrow a} (f(x) - g(x)) = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$	

Soal-soal:

- a. $\lim_{x \rightarrow 6} 25 = 25$ b. $\lim_{x \rightarrow 0} 36 = 36$ c. $\lim_{x \rightarrow -2} 9 = 9$
- $\lim_{x \rightarrow 3} x^4 = 3^4 = 81$
- $\lim_{x \rightarrow 2} x^3 - 5x + 7 = 2^3 - 5 \cdot 2 + 7 = 5$

- 4). $\lim_{x \rightarrow -2} 5x = 5 \lim_{x \rightarrow -2} x = 5 \cdot (-2) = -10$
- 5). $\lim_{x \rightarrow 4} 5x + 3x^2 = \lim_{x \rightarrow 4} 5x + \lim_{x \rightarrow 4} 3x^2 = 5 \cdot 4 + 3 \cdot 4^2 = 20 + 48 = 68$
- 6). $\lim_{x \rightarrow 4} 5x - 3x^2 = \lim_{x \rightarrow 4} 5x - \lim_{x \rightarrow 4} 3x^2 = 5 \cdot 4 - 3 \cdot 4^2 = 20 - 48 = -28$
- 7). $\lim_{x \rightarrow 1} (5x + 3x^2)(5x - 1) = \lim_{x \rightarrow 1} (5x + 3x^2) \lim_{x \rightarrow 1} (5x - 1) = 8 \cdot 4 = 32$
- 8). $\lim_{x \rightarrow 1} \frac{(5x + 3x^2)}{(5x - 1)} = \frac{\lim_{x \rightarrow 1} (5x + 3x^2)}{\lim_{x \rightarrow 1} (5x - 1)} = \frac{8}{4} = 2$
- 9). $\lim_{x \rightarrow 1} (5x + 2)^3 = \left(\lim_{x \rightarrow 1} (5x + 2) \right)^3 = (5 \cdot 1 + 2)^3 = 7^3 = 343$
- 10). $\lim_{x \rightarrow 1} \sqrt[3]{5x + 2} = \sqrt[3]{\lim_{x \rightarrow 1} (5x + 2)} = \sqrt[3]{(5 \cdot 1 + 2)} = \sqrt[3]{7}$
- 11). $\lim_{x \rightarrow -5} \frac{5x - 3x^2}{2x + 7} = \frac{\lim_{x \rightarrow -5} (5x - 3x^2)}{\lim_{x \rightarrow -5} (2x + 7)} = \frac{\lim_{x \rightarrow -5} 5x - \lim_{x \rightarrow -5} 3x^2}{\lim_{x \rightarrow -5} 2x + \lim_{x \rightarrow -5} 7} = \frac{5 \cdot (-5) - 3 \cdot (-5)^2}{2 \cdot (-5) + 7} = \frac{-25 - 75}{-10 + 7} = \frac{-100}{-3} = \frac{100}{3}$

4. Limit Fungsi Trigonometri

Cara menentukan nilai limit fungsi trigonometri sama dengan limit fungsi aljabar.

Beberapa persamaan khusus:

a. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$	d. $\lim_{x \rightarrow 0} \frac{\tan ax}{bx} = \lim_{x \rightarrow 0} \frac{ax}{\tan bx} = \frac{a}{b}$
b. $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{x}{\tan x} = 1$	e. $\lim_{x \rightarrow 0} \frac{\tan ax}{\sin bx} = \lim_{x \rightarrow 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}$
c. $\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \lim_{x \rightarrow 0} \frac{ax}{\sin bx} = \frac{a}{b}$	

Soal-soal:

- 1). $\lim_{x \rightarrow 0} \frac{x}{\cos x} = \frac{0}{\cos 0} = \frac{0}{1} = 0$
- 2). $\lim_{x \rightarrow \frac{1}{2}\pi} \sin x + \cos x = \sin \frac{1}{2}\pi + \cos \frac{1}{2}\pi = 1 + 0 = 1$
- 3). $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \lim_{x \rightarrow 0} \frac{\sin 2x}{x} \cdot \frac{2}{2} = 2 \cdot \lim_{2x \rightarrow 0} \frac{\sin 2x}{2x} = 2 \cdot 1 = 2$ (jika $x \rightarrow 0$ maka $2x \rightarrow 0$)
- 4). $\lim_{x \rightarrow 0} \frac{3x + \sin 4x}{5x - \tan 2x} = \frac{0}{0}$ BTT, maka (khusus soal model ini, pembilang dan penyebut dibagi dengan x)
- $$\lim_{x \rightarrow 0} \frac{3x + \sin 4x}{5x - \tan 2x} = \lim_{x \rightarrow 0} \frac{\frac{3x}{x} + \frac{\sin 4x}{x}}{\frac{5x}{x} - \frac{\tan 2x}{x}} = \lim_{x \rightarrow 0} \frac{3 + \frac{\sin 4x}{x}}{5 - \frac{\tan 2x}{x}} = \frac{\lim_{x \rightarrow 0} 3 + \lim_{x \rightarrow 0} \frac{\sin 4x}{x}}{\lim_{x \rightarrow 0} 5 - \lim_{x \rightarrow 0} \frac{\tan 2x}{x}} = \frac{3 + 4}{5 - 2} = \frac{7}{3}$$
- 5). $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x \sin x} = \frac{0}{0}$ BTT, maka
- $$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x \sin x} &= \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x \sin x} \cdot \frac{1 + \cos 4x}{1 + \cos 4x} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 4x}{(x \sin x)(1 + \cos 4x)} = \lim_{x \rightarrow 0} \frac{\sin^2 4x}{(x \sin x)(1 + \cos 4x)} \\ &= \lim_{x \rightarrow 0} \frac{\sin 4x \cdot \sin 4x}{x \sin x} \cdot \frac{1}{1 + \cos 4x} \cdot \frac{4x \cdot 4x}{4x \cdot 4x} = \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \cdot \frac{\sin 4x}{4x} \cdot \frac{4x}{\sin x} \cdot \frac{1}{(1 + \cos 4x)} \cdot \frac{4x}{x} \\ &= 1 \cdot 1 \cdot 4 \cdot \frac{1}{2} \cdot 4 = 8 \end{aligned}$$

$$6). \lim_{x \rightarrow \frac{p}{2}} \left(\frac{\cos x}{x - \frac{p}{2}} \right) = \frac{0}{0} \text{ BTT, maka}$$

Diketahui rumus trigonometri: $\cos x = \sin \left(\frac{p}{2} - x \right)$

$$\lim_{x \rightarrow \frac{p}{2}} \left(\frac{\cos x}{x - \frac{p}{2}} \right) = \lim_{x \rightarrow \frac{p}{2}} \frac{\sin \left(\frac{p}{2} - x \right)}{x - \frac{p}{2}} = \lim_{x \rightarrow \frac{p}{2}} \frac{\sin - \left(x - \frac{p}{2} \right)}{x - \frac{p}{2}} = \lim_{x \rightarrow \frac{p}{2}} \frac{-\sin \left(x - \frac{p}{2} \right)}{x - \frac{p}{2}} = - \lim_{x \rightarrow \frac{p}{2}} \frac{\sin \left(x - \frac{p}{2} \right)}{x - \frac{p}{2}} = -1$$

5. Kekontinuan Suatu Fungsi

Suatu fungsi f dikatakan kontinu pada $x = a$ jika:

- $f(a)$ ada
- $\lim_{x \rightarrow a} f(x)$ ada
- $\lim_{x \rightarrow a} f(x) = f(a)$

Ciri:
 Grafiknya merupakan lengkungan (kurva)
 yang tidak terputus.

Soal-soal:

1). Fungsi $f(x) = 2x + 1$, kontinu di $x = 1$ karena $\lim_{x \rightarrow 1} (2x + 1) = 3 = f(1)$

2). Fungsi $f(x) = \begin{cases} x^2 - 9 & ; x \neq 3 \\ 3 & ; x = 3 \end{cases}$ maka $f(x)$ tidak kontinu di $x = 3$, karena

a. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{(x - 3)} = \lim_{x \rightarrow 3} (x + 3) = 3 + 3 = 6$

b. $f(3) = 3$

maka $\lim_{x \rightarrow 3} f(x) \neq f(3)$

6. Limit Barisan Bilangan

a. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e$

b. $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^{\frac{1}{n}} = e$

c. $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n} \right)^n = e^{-1}$

d. $\lim_{n \rightarrow \infty} (1 - \frac{1}{n})^{\frac{1}{n}} = e^{-1}$

Ket: $e = 2,7182818... = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + ...$ (bilangan Euler)

Soal-soal:

1. $\lim_{x \rightarrow \infty} \left(\frac{x}{x+1} \right)^{x+1} = \lim_{x \rightarrow \infty} \left(\frac{x+1-1}{x+1} \right)^{x+1} = \lim_{x \rightarrow \infty} \left(\frac{x+1}{x+1} - \frac{1}{x+1} \right)^{x+1} = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x+1} \right)^{x+1} = e^{-1}$

Persamaan c.

atau $\lim_{x \rightarrow \infty} \left(\frac{x}{x+1} \right)^{x+1} = \lim_{x \rightarrow \infty} \left(\frac{x+1-1}{x+1} \right)^{x+1} = \lim_{x \rightarrow \infty} \left(\frac{x+1}{x+1} - \frac{1}{x+1} \right)^{x+1} = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x+1} \right)^{x+1}$

atau

$$= \lim_{x \rightarrow \infty} \left\{ \left(1 - \frac{1}{x+1} \right)^{-(x+1)} \right\}^{-1} = \lim_{x \rightarrow \infty} \left\{ \left(1 + \frac{1}{-(x+1)} \right)^{-(x+1)} \right\}^{-1} = e^{-1}$$

Persamaan a.

2. $\lim_{x \rightarrow \infty} (1 - 3x)^{\frac{1}{x}} = \lim_{x \rightarrow \infty} (1 - 3x)^{\frac{-3}{3x-1}} = \lim_{x \rightarrow \infty} \left[(1 - 3x)^{\frac{-1}{3x}} \right]^{-3} = \left[\lim_{x \rightarrow \infty} (1 - 3x)^{\frac{-1}{3x}} \right]^{-3} = e^{-3}$

$$\begin{aligned}
 3. \lim_{x \rightarrow \infty} \left(1 + \frac{2}{3+x}\right)^{-2x} &= \lim_{x \rightarrow \infty} \left(1 + \frac{2}{3+x}\right)^{\left(\frac{3+x}{2}\right)^{(-4)+6}} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{2}{3+x}\right)^{\left(\frac{3+x}{2}\right)} \right]^{-4} \left[\left(1 + \frac{2}{3+x}\right)^6 \right] \\
 &= \left[\lim_{x \rightarrow \infty} \left(1 + \frac{2}{3+x}\right)^{\left(\frac{3+x}{2}\right)} \right]^{-4} \cdot \lim_{x \rightarrow \infty} \left(1 + \frac{2}{3+x}\right)^6 = e^{-4} \cdot (1+0)^6 = e^{-4}
 \end{aligned}$$

Catatan:

- a. $a^2 - b^2 = (a-b)(a+b)$
- b. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- c. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- d. $(a+b)^2 = a^2 + 2ab + b^2$
- e. $(a-b)^2 = a^2 - 2ab + b^2$
- f. $(\sqrt{a})^2 = \sqrt{a} \cdot \sqrt{a} = a$
- g. $(\sqrt{a+b})^2 = \sqrt{a+b} \cdot \sqrt{a+b} = a+b$

Bentuk Sekawan:

- a. $\sqrt{a} - \sqrt{b}$ sekawannya $\sqrt{a} + \sqrt{b}$
- b. $a + \sqrt{b-c}$ sekawannya $a - \sqrt{b-c}$
- c. $a\sqrt{b-c}$ sekawannya $a\sqrt{b+c}$
- d. $\sqrt{a+b} + \sqrt{c-d}$ sekawannya $\sqrt{a+b} - \sqrt{c-d}$
- e. $\sqrt{a+b} - c$ sekawannya $\sqrt{a+b} + c$

dan lain sebagainya..

Catatan 2:

- a. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
- b. $\frac{\sqrt{a}}{x} = \frac{\sqrt{a}}{\sqrt{x^2}} = \sqrt{\frac{a}{x^2}}$
- c. $\frac{\sqrt{a+b}}{x^2} = \frac{\sqrt{a+b}}{\sqrt{x^4}} = \sqrt{\frac{a+b}{x^4}} = \sqrt{\frac{a}{x^4} + \frac{b}{x^4}}$
- d. $\frac{\sqrt{ax^2+bx}}{x^3} = \frac{\sqrt{ax^2+bx}}{\sqrt{x^6}} = \sqrt{\frac{ax^2+bx}{x^6}} = \sqrt{\frac{ax^2}{x^6} + \frac{bx}{x^6}}$

dan lain-lain.

Keterangan:

Sebagian materi adalah materi pengayaan, tidak semuanya dipelajari di kelas.

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Soal-Soal Latihan



A. Kerjakan soal-soal berikut, bila perlu gambarlah grafiknya.

1. Jika $f(x) = \begin{cases} 2; & jk \ x \leq 0 \\ x^2; & jk \ x > 0 \end{cases}$, tentukan: a. $\lim_{x \rightarrow 0^-} f(x)$, b. $\lim_{x \rightarrow 0^+} f(x)$, c. $\lim_{x \rightarrow 0} f(x)$ jk ada.

2. Jika $f(x) = \begin{cases} 3x+2; & jk \ x < 1 \\ x+4; & jk \ x \geq 1 \end{cases}$, tentukan: a. $\lim_{x \rightarrow 1^-} f(x)$, b. $\lim_{x \rightarrow 1^+} f(x)$, c. $\lim_{x \rightarrow 1} f(x)$.

3. Jika $f(x) = \begin{cases} 4x+1; & jk \ x \leq 1 \\ 2x^2+3; & jk \ x > 1 \end{cases}$, tentukan: a. $\lim_{x \rightarrow 1^-} f(x)$, b. $\lim_{x \rightarrow 1^+} f(x)$, c. $\lim_{x \rightarrow 1} f(x)$.

4. Jika $f(x) = \begin{cases} -1; & jk \ x < -1 \\ 0; & jk \ x = -1 \\ 1; & jk \ x > -1 \end{cases}$, tentukan: a. $\lim_{x \rightarrow -1^-} f(x)$, b. $\lim_{x \rightarrow -1^+} f(x)$, c. $\lim_{x \rightarrow -1} f(x)$.

5. Ditentukan $f(x) = \begin{cases} 2; & jk \ x < -1 \\ 1-x; & jk \ -1 \leq x < 1 \\ 0; & jk \ x \geq 1 \end{cases}$

Selidiki apakah ada nilai limit fungsi berikut: a. $\lim_{x \rightarrow -1} f(x)$ b. $\lim_{x \rightarrow 1} f(x)$

6. Tentukan nilai dari: a. $\lim_{x \rightarrow 1^+} \sqrt{x-1}$ b. $\lim_{x \rightarrow -1^+} x^2$ c. $\lim_{x \rightarrow 0^+} \frac{1}{x^2}$

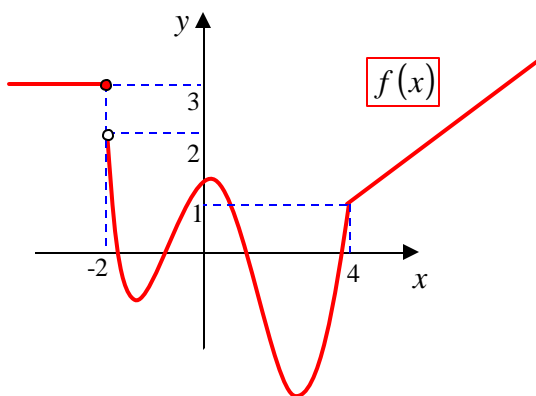
7. Tentukan nilai dari: a. $\lim_{x \rightarrow 4^-} \sqrt{4x}$ b. $\lim_{x \rightarrow -2^-} x$ c. $\lim_{x \rightarrow 0^-} \frac{3}{2x}$

8. Diketahui fungsi $f(x) = \sqrt{x}$. Tentukan nilai berikut jika ada! (cari limit kiri dan limit kanan).

a. $\lim_{x \rightarrow 1} f(x)$ b. $\lim_{x \rightarrow 3} f(x)$ c. $\lim_{x \rightarrow 16} f(x)$ d. $\lim_{x \rightarrow 0} f(x)$

9. Selidikilah, apakah $\lim_{x \rightarrow 0} \frac{1}{x}$ ada? (cari limit kiri dan limit kanan).

10. Tentukan $\lim_{x \rightarrow -2} f(x)$ dan $\lim_{x \rightarrow 4} f(x)$ dari gambar berikut:



B. Hitunglah nilai dari limit fungsi berikut:

11. $\lim_{x \rightarrow 5} 1000$

12. $\lim_{x \rightarrow 1} 12345$

13. $\lim_{x \rightarrow -2} 2x + 5$

14. $\lim_{x \rightarrow 0} 3x^2 + 5x - 10$

15. $\lim_{x \rightarrow -3} (x-4)(x+1)$

16. $\lim_{x \rightarrow -5} [(4x-7)\sqrt[3]{3-x}]$

17. $\lim_{x \rightarrow 4} \frac{x}{x+2}$

18. $\lim_{x \rightarrow 4} \left[\left(\frac{3x-1}{x+2} \right) \sqrt{\frac{x}{x-3}} \right]$

19. $\lim_{x \rightarrow 0} \frac{3x^2 - 5x + 10}{x^3 + 6x - 45}$

20. $\lim_{x \rightarrow 2} \frac{6x+9}{7x-10}$

21. $\lim_{x \rightarrow 9} \sqrt{4x-11}$

22. $\lim_{x \rightarrow 4} \sqrt{x^2-7}$

23. $\lim_{x \rightarrow 1} \sqrt{\frac{x^2-6}{-x^3}}$

24. $\lim_{x \rightarrow 2} \sqrt{\frac{x^2+3x+6}{x^3+1}}$

25. $\lim_{x \rightarrow 2} \frac{1}{x-2}$

26. $\lim_{x \rightarrow 4} \frac{x+4}{x^2-2x-24}$

27. $\lim_{x \rightarrow -1} \frac{\sqrt{x+5}}{x^2-2x-24}$

28. $\lim_{x \rightarrow -3} \frac{\sqrt{6-x}}{\sqrt{x+6}}$

29. $\lim_{x \rightarrow 3} \frac{x-3}{x}$

30. $\lim_{x \rightarrow 2} \left(\frac{2x-3}{x} + \frac{2x}{6-7x} \right)$

31. $\lim_{x \rightarrow -2} \left(\frac{9x}{8+5x} + \sqrt{5x+14} \right)$

32. $\lim_{x \rightarrow 5} \frac{(x-3)(x-5)}{2x-1}$

33. $\lim_{x \rightarrow 7} \frac{(x-3)(x-5)}{\sqrt{2x+2}+5x}$

34. $\lim_{x \rightarrow 1} \frac{\sqrt{x+3}+\sqrt{5x+4}}{\sqrt{15-6x}-\sqrt{2x-1}}$

35. $\lim_{x \rightarrow -4} (\sqrt{8-2x}+\sqrt{-5x+5})$

36. $\lim_{x \rightarrow 3} (\sqrt{2x^2+3x-2}-\sqrt{2x^2-4x+3})$

37. $\lim_{x \rightarrow a} \frac{x+9}{2x-1}$

38. $\lim_{x \rightarrow m} \frac{7x}{m}$

39. $\lim_{x \rightarrow n} \frac{x^2+x}{n}$

40. Jika $\lim_{x \rightarrow n} (x+1) = \lim_{x \rightarrow n} (2x-3)$, maka tentukan nilai dari: $\lim_{x \rightarrow n} (x^2-16)$

41. Jika $\lim_{x \rightarrow 7} \frac{x^2-6x-7}{x^2-10x+21} = a$, berapakah nilai dari $\lim_{x \rightarrow a} \frac{4x^2-7x-2}{3-\sqrt{4x+1}}$?

42. Jika $\lim_{x \rightarrow -2} \frac{2x^2+5x+2}{x^2+ax-10} = \frac{3}{7}$, maka $a = \dots$

43. Jika $\lim_{x \rightarrow 3} \frac{3x^2+ax-1}{x^2-ax-30} = \frac{11}{13}$, maka $a = \dots$

44. $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$

45. $\lim_{x \rightarrow 1} \frac{x-1}{1-\sqrt{x}}$

46. $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$

47. $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{1-x}$

48. $\lim_{x \rightarrow 1} \frac{x^2+5x-6}{x-1}$

49. $\lim_{x \rightarrow -3} \frac{2x+6}{x^2+x-6}$

$$50. \lim_{x \rightarrow 0} \frac{3x^2 - 5x}{x}$$

$$51. \lim_{x \rightarrow 0} \frac{\sqrt{x}}{x + \sqrt{x}}$$

$$52. \lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$$

53. Dengan menyederhanakan lebih dahulu (menyamakan penyebut), hitunglah:

$$a. \lim_{x \rightarrow 0} \left(\frac{1}{x^2 - x} + \frac{1}{x} \right)$$

$$b. \lim_{x \rightarrow 0} \left(\frac{2}{x^2 - 1} - \frac{1}{x - 1} \right)$$

$$c. \lim_{x \rightarrow 1} \left(\frac{1}{1 - x} - \frac{3}{1 - x^3} \right)$$

$$d. \lim_{x \rightarrow 2} \left(\frac{2}{x^2 - 4} - \frac{3}{x^2 + 2x - 8} \right)$$

$$54. \lim_{x \rightarrow -1} \frac{2x + 2}{x^2 - 3x - 4}$$

$$55. \lim_{x \rightarrow 2} \frac{3x^2 - 6x}{x - 2}$$

$$56. \lim_{x \rightarrow 3} \frac{(x - 2)^2 - 1}{x - 3} \text{ (Ebtanas IPS 99)}$$

$$57. \lim_{x \rightarrow \frac{1}{2}} \frac{2x - 1}{2x^2 + 3x - 2}$$

$$58. \lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x^2 - 2x + 1}$$

$$59. \lim_{x \rightarrow 0} \frac{x^2 + 2x}{x^3 + x^2 + 3x}$$

$$60. \lim_{x \rightarrow 0} \frac{x^4 - 6x^2}{x^3 + 2x^2}$$

$$61. \lim_{x \rightarrow 0} \frac{x^{n+3} + 6x^{n+1} - x^n}{x^{n+4} + 2x^n}$$

$$62. \lim_{x \rightarrow 1} \frac{2x^3 + 3x^2 - 2x - 3}{x^2 - 1}$$

$$63. \lim_{x \rightarrow 2} \frac{x^3 + x^2 - 8x + 4}{x^3 - 2x^2 - x + 2}$$

$$64. \lim_{x \rightarrow 2} \frac{x^3 + x^2 - 6x}{x^3 - 2x^2 + 6x - 12}$$

$$65. \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$$

$$66. \lim_{x \rightarrow 1} \frac{x^3 - 1}{1 - x}$$

$$67. \lim_{x \rightarrow 3} \frac{x - 3}{x^3 - 27}$$

$$68. \lim_{x \rightarrow 4} \frac{4 - x}{x^3 - 64}$$

$$69. \lim_{x \rightarrow 1} \frac{x - 1}{\sqrt[3]{x} - 1}$$

$$70. \lim_{x \rightarrow \frac{3}{2}} \sqrt{\frac{8x^3 - 27}{4x^2 - 9}}$$

$$71. \lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{\sqrt{x} - 2}$$

$$72. \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x^4 - x}$$

73. Diketahui $g(x) = \sqrt{1 + 2x}$, maka nilai $\lim_{x \rightarrow 0} \frac{g(1+x) - g(1-x)}{x} = \dots$

$$74. \lim_{x \rightarrow 1} \frac{x - 1}{2 - \sqrt{3x + 1}}$$

$$75. \lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{\sqrt{2x + 5} - \sqrt{x + 7}}$$

$$76. \lim_{x \rightarrow 6} \frac{\sqrt{x - 2} - \sqrt{10 - x}}{\sqrt{6x} - \sqrt{5x + 6}}$$

$$77. \lim_{x \rightarrow 3} \frac{\sqrt{x + 2} - \sqrt{2x - 1}}{\sqrt{2x - 3} - \sqrt{x}}$$

$$78. \lim_{x \rightarrow 1} \frac{\sqrt{3 - x} - \sqrt{3x - 1}}{\sqrt{5x - 1} - \sqrt{x + 3}}$$

$$79. \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 2x + 3} - \sqrt{x^2 - 2x + 3}}{\sqrt{x + 3} - \sqrt{3 - x}}$$

$$80. \lim_{x \rightarrow 3} \frac{\sqrt{5x + 1} - 4}{x^2 - 9}$$

$$81. \lim_{x \rightarrow 10} \frac{\sqrt{x - 1} - 3}{x - 10}$$

$$82. \lim_{x \rightarrow 3} \frac{x - \sqrt{2x + 3}}{x^2 - 9}$$

$$83. \lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 3} - x - 1}{1 - x^2}$$

**

84. $\lim_{x \rightarrow 3} \frac{9 - x^2}{4 - \sqrt{x^2 + 7}}$
85. $\lim_{x \rightarrow 0} \frac{2x^2 - 5x}{3 - \sqrt{9 + x}}$
86. $\lim_{x \rightarrow 5} \frac{4 - \sqrt{x^2 - 9}}{5 - x}$
87. $\lim_{x \rightarrow 3} \frac{\sqrt{x+4} - \sqrt{2x+1}}{x-3}$
88. $\lim_{x \rightarrow 5} \frac{\sqrt{x+4} - \sqrt{x-4}}{\sqrt{x} - \sqrt{5}}$
89. $\lim_{x \rightarrow 2} \frac{\sqrt{x} - 2}{\sqrt{2x+1} + \sqrt{2-x}}$
90. $\lim_{x \rightarrow 1} \frac{\sqrt{3+x} + \sqrt{5x-1}}{\sqrt{3+x} - \sqrt{5x-1}}$
91. $\lim_{x \rightarrow 2} \frac{2x - \sqrt{x+3}}{\sqrt{3x+6} - x}$
92. $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{\sqrt{3-x} - \sqrt{x-3}}$
93. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$
94. $\lim_{x \rightarrow 0} \frac{4x}{\sqrt{1+2x} - \sqrt{1-2x}}$
95. $\lim_{x \rightarrow 1} \frac{1-x}{\sqrt{1-x} - \sqrt{x-1}}$
96. $\lim_{x \rightarrow 0} \frac{x^2}{1 - \sqrt[3]{1+x^2}}$
97. $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x^2} - 2\sqrt[3]{x} - 1}{(x-1)^2}$ **
98. $\lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1}$ **
99. Diketahui $f(x) = 3x^2 - 2x$, tentukan
- $$\lim_{x \rightarrow 2} \frac{\left(f(x) - \frac{1}{4} \cdot f(2)(x+2) \right)}{x-2}$$
100. Diketahui $f(x) = \frac{3}{x^2}$, tentukan
- $$\lim_{x \rightarrow 2} \frac{(f(x) - f(2))}{x-2}$$

C. Hitunglah nilai dari limit fungsi berikut:

101. $\lim_{x \rightarrow \infty} \frac{2}{x}$
102. $\lim_{x \rightarrow \infty} \frac{6}{5x^{10}}$
103. $\lim_{x \rightarrow \infty} \frac{-9}{2x^{25}}$
104. $\lim_{x \rightarrow \infty} \frac{7}{2x^3 + 5x}$
105. $\lim_{x \rightarrow \infty} \frac{-3}{x^3 - 20}$
106. $\lim_{x \rightarrow \infty} 4x + 99$
107. $\lim_{x \rightarrow \infty} x^2 + 9x - 15$
108. $\lim_{x \rightarrow \infty} \frac{3x}{100}$
109. $\lim_{x \rightarrow \infty} \frac{7x + 4}{55}$
110. $\lim_{x \rightarrow \infty} \frac{x^2 - 25}{12}$
111. $\lim_{x \rightarrow \infty} \frac{x + 5}{2x - 1}$
112. $\lim_{x \rightarrow \infty} \frac{4x - 3}{2x + 5}$
113. $\lim_{x \rightarrow \infty} \frac{6 - 8x}{x + 5}$
114. $\lim_{x \rightarrow \infty} \frac{10 + 3x}{9x - 5}$
115. $\lim_{x \rightarrow \infty} \frac{10 + 3x}{3 - 9x}$
116. $\lim_{x \rightarrow \infty} \frac{7 - 5x^2}{3x + 12x^2}$
117. $\lim_{x \rightarrow \infty} \frac{5x^3 - 11x^2}{3x + 12x^3}$
118. $\lim_{x \rightarrow \infty} \frac{(5x-1)(2x+3)}{(3+12x)(x-1)}$
119. $\lim_{x \rightarrow \infty} \frac{x^2 + 5x - 3}{(3-x)(x-1)}$
120. $\lim_{x \rightarrow \infty} \frac{(x-1)(x-3)}{2x^2 + 3x - 15}$

121. $\lim_{x \rightarrow \infty} \frac{4(2x+3)^3}{3x^3+5x}$
122. $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^4+8x}}{2x^2}$
123. $\lim_{x \rightarrow \infty} \frac{4x^2+3x-1}{3x^2+5x-2}$
124. $\lim_{x \rightarrow \infty} \frac{x+3x^3}{3x^3-2}$
125. $\lim_{x \rightarrow \infty} \frac{(2x-5)^4}{(3x^2+2)^2}$
126. $\lim_{x \rightarrow \infty} \frac{6x+x^3-5x^4}{x^3-2x^4}$
127. $\lim_{x \rightarrow \infty} \frac{x(2x+1)^2}{5x-4x^3}$
128. $\lim_{x \rightarrow \infty} \frac{2(x-1)^3}{x^3+1}$
129. $\lim_{x \rightarrow \infty} \frac{6x+2x^3}{(x-3)(x+1)}$
130. $\lim_{x \rightarrow \infty} \frac{(x^2-2)(x^2+2)}{x(x-1)(x+1)}$
131. $\lim_{x \rightarrow \infty} \frac{2x^3+7x-5}{x^2-x}$
132. $\lim_{x \rightarrow \infty} \frac{2x^2+x}{\sqrt{6x+3x^3}}$
133. $\lim_{x \rightarrow \infty} \frac{\sqrt{2x+x^4}}{2x-3}$
134. $\lim_{x \rightarrow \infty} \frac{9x^4+x}{x^2-x^3}$
135. $\lim_{x \rightarrow \infty} \frac{3x^2-5}{2x^3+x-1}$
136. $\lim_{x \rightarrow \infty} \frac{3x+5}{2x^2+4x+5}$
137. $\lim_{x \rightarrow \infty} \frac{3x^2+5x-7}{10x^3+5x}$
138. $\lim_{x \rightarrow \infty} \frac{x^2-17}{\sqrt{x^6+5x^3-5}}$
139. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+5x-1}}{3x^2-9}$
140. $\lim_{x \rightarrow \infty} \left(\frac{\sqrt{x+4}-\sqrt{2x+1}}{x-3} \right)$
141. $\lim_{x \rightarrow \infty} \frac{x^2-17}{\sqrt{x^6+5x^3-5}+\sqrt{3x^6-2}}$
142. $\lim_{x \rightarrow \infty} \frac{x-2}{\sqrt{4x^2-2x-6}-\sqrt{x^2+1}}$
143. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+5x-1}}{\sqrt{3x^4-9x+1}}$
144. $\lim_{x \rightarrow \infty} (\sqrt{x+6}-\sqrt{x+3})$
145. $\lim_{x \rightarrow \infty} (\sqrt{x+3}-\sqrt{x+2})$
146. $\lim_{x \rightarrow \infty} (\sqrt{2x-1}-\sqrt{x+4})$
147. $\lim_{x \rightarrow \infty} (\sqrt{4x+2}-\sqrt{x-3})$
148. $\lim_{x \rightarrow \infty} (\sqrt{x+5}-\sqrt{x})$
149. $\lim_{x \rightarrow \infty} (\sqrt{3x+1}-\sqrt{3x-1})$
150. $\lim_{x \rightarrow \infty} (\sqrt{x+1}-2\sqrt{x-3})$
151. $\lim_{x \rightarrow \infty} (3\sqrt{x+6}-2\sqrt{1-x})$
152. $\lim_{x \rightarrow \infty} (\sqrt{ax+b}-\sqrt{px+q})$
 untuk: $a = p$, $a > p$ dan $a < p$
153. $\lim_{x \rightarrow \infty} (\sqrt{x^2+x+1}-\sqrt{2x^2+x})$
154. $\lim_{x \rightarrow \infty} (\sqrt{4x^2+6x-1}-\sqrt{5x^2-x+9})$
155. $\lim_{x \rightarrow \infty} (\sqrt{x^2+2x-1}-\sqrt{(x-2)(2x+9)})$
156. $\lim_{x \rightarrow \infty} (\sqrt{4x^2-5}-\sqrt{x^2-3x})$
157. $\lim_{x \rightarrow \infty} (\sqrt{2x^2+x-5}-\sqrt{x^2-3x+12})$
158. $\lim_{x \rightarrow \infty} (\sqrt{(3x+1)(x-5)}-\sqrt{x^2+7x+1})$
159. $\lim_{x \rightarrow \infty} (\sqrt{(3x-5)(x+4)}-\sqrt{3x^2-7x+1})$
160. $\lim_{x \rightarrow \infty} (x-\sqrt{4x^2-7x-1})$
161. $\lim_{x \rightarrow \infty} ((x+2)-\sqrt{4x^2-7x+8})$
162. $\lim_{x \rightarrow \infty} (x+5-\sqrt{x^2-x-9})$
163. $\lim_{x \rightarrow \infty} ((x+3)-\sqrt{(x-3)(x+3)})$

$$164. \lim_{x \rightarrow \infty} (\sqrt{3x^2 + 3x - 5} - x + 4)$$

$$165. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 6x + 5} - x - 4)$$

$$166. \lim_{x \rightarrow \infty} (\sqrt{x^2 - 1} - 2x - 3)$$

$$167. \lim_{x \rightarrow \infty} (\sqrt{4x^2 + 3x - 5} - (2x - 3))$$

$$168. \lim_{x \rightarrow \infty} (\sqrt{9x^2 + x - 4} - (3x + 5))$$

$$169. \lim_{x \rightarrow \infty} (\sqrt{2x^2 - 3x + 5})$$

$$170. \lim_{x \rightarrow \infty} (\sqrt{x^2 - 3x} - \sqrt{2x^2 + 8})$$

$$171. \lim_{x \rightarrow \infty} (\sqrt{3x - \sqrt{x}} - 4 - \sqrt{3x + 2\sqrt{x} - 5})$$

$$172. \lim_{x \rightarrow \infty} (\sqrt{4x^4 + 3x^2 - 1} - \sqrt{4x^4 + 5x^2 + 1})$$

$$173. \lim_{x \rightarrow \infty} (\sqrt{x^3 - 4} - \sqrt{x^3 + 8})$$

$$174. \lim_{x \rightarrow \infty} (x(\sqrt{x^2 + 2} - x))$$

$$175. \lim_{x \rightarrow \infty} \left(\frac{4}{x^2} - \frac{3}{x} + 2 \right)$$

$$176. \lim_{x \rightarrow \infty} \frac{\sqrt[3]{x^2} + x\sqrt[3]{x}}{x^2} \quad **$$

$$177. \lim_{x \rightarrow \infty} \frac{3\sqrt[3]{x^2} - x^2\sqrt{x}}{6 + x^2\sqrt{x}} \quad **$$

$$185. \lim_{x \rightarrow 0} \frac{x \sin 5x}{\sin^2 3x}$$

$$186. \lim_{x \rightarrow 0} \frac{\tan^2 \frac{1}{2} x}{\sin 3x \sin 2x}$$

$$187. \lim_{x \rightarrow 0} \frac{2x^2}{\sin^2 x}$$

$$188. \lim_{x \rightarrow 0} \frac{\sin^2 3x}{(3x)^2}$$

$$189. \lim_{x \rightarrow 0} \frac{\tan 2x}{x \sec 2x}$$

$$190. \lim_{x \rightarrow 0} \frac{x}{\sin \frac{x}{2} \cos \frac{x}{2}}$$

$$191. \lim_{x \rightarrow 0} \frac{2x}{\cos x}$$

$$192. \lim_{x \rightarrow 0} \frac{\sin^2 2x}{x^2}$$

$$193. \lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{x^2}$$

$$194. \lim_{x \rightarrow 0} \frac{\sin 3x + \sin 4x}{x}$$

$$195. \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x}$$

$$196. \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{2x^2}$$

$$197. \lim_{x \rightarrow 0} \frac{\sin(2x^2)}{x^2 + \sin^2 3x}$$

$$198. \lim_{x \rightarrow 0} \frac{\sin 4x \tan^2 3x + 6x^3}{2x^2 \sin 3x \cos 2x}$$

$$199. \lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a}$$

$$200. \lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{1 - \cos x}$$

$$201. \lim_{x \rightarrow \frac{p}{4}} \frac{\cos 2x}{p - 4x}$$

$$202. \lim_{x \rightarrow \frac{p}{4}} \frac{\cos^2 x}{1 - \sin x}$$

$$203. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin x}$$

D. Hitunglah nilai dari limit fungsi berikut:

$$178. \lim_{x \rightarrow \frac{p}{2}} \sin x + 5 \cos x$$

$$179. \lim_{x \rightarrow 0} (\sin 2x \cdot \cot x)$$

$$180. \lim_{x \rightarrow \frac{p}{2}} \left(\frac{\sin x}{6} + \frac{5 \cos x}{3 \sin x} \right)$$

$$181. \lim_{x \rightarrow 0} \frac{\cos x}{2x}$$

$$182. \lim_{x \rightarrow 0} \frac{x + 5}{\cos x}$$

$$183. \lim_{x \rightarrow 0} \frac{\tan 2x}{\sin 5x}$$

$$184. \lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$$

204. $\lim_{x \rightarrow \frac{p}{2}} (\sec x - \tan x)$
205. $\lim_{x \rightarrow 0} \frac{\sin x - \tan x}{x^3}$
206. $\lim_{x \rightarrow 0} (x \cot 2x)$
207. $\lim_{x \rightarrow 1} \frac{x-1}{\tan px}$
208. $\lim_{x \rightarrow \frac{p}{4}} \frac{\tan x - 1}{\cos 2x}$
209. $\lim_{x \rightarrow \frac{p}{4}} \frac{\cos 2x}{x(\tan x - 1)}$
210. $\lim_{x \rightarrow 0} \frac{\sin 2x}{3 - \sqrt{2x+9}}$
211. $\lim_{x \rightarrow 0} \frac{\sin 4x}{1 - \sqrt{1-x}}$
212. $\lim_{x \rightarrow 2} \frac{\sin(x-2)}{x-2}$
213. $\lim_{x \rightarrow p} \frac{\sin(x-p)}{x-p}$
214. $\lim_{x \rightarrow 1} \frac{(3x+1)\sin(x-1)}{x^2 + 2x - 3}$
215. $\lim_{x \rightarrow 3} \frac{\sin(\sqrt{x+1}-2)}{x-3}$
216. $\lim_{x \rightarrow \frac{p}{2}} \frac{1 - \sin x}{\frac{p}{2} - x}$
217. $\lim_{x \rightarrow \frac{p}{2}} \frac{2 \tan x}{\sec x}$
218. $\lim_{x \rightarrow 0} \frac{\tan 2x \cdot \tan 3x}{5x^2}$
219. $\lim_{x \rightarrow 0} \frac{1 + \cos x}{1 + \sin x}$
220. $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{1 - \cos x}$
221. $\lim_{x \rightarrow 0} \frac{x^2 + 3x}{\sin x}$
222. $\lim_{x \rightarrow 0} \frac{2x^2}{1 - \cos^2 \frac{1}{2}x}$
223. $\lim_{x \rightarrow 0} \frac{\sin 3x - \sin 3x \cdot \cos 2x}{4x^3}$
224. $\lim_{x \rightarrow 2} \frac{(x^2 - 5x + 6)\sin(x-2)}{(x^2 - x - 2)^2}$
225. $\lim_{x \rightarrow 0} \frac{(x^2 - 1)\sin 6x}{x^3 + 3x^2 + 2x}$
226. $\lim_{x \rightarrow 0} \frac{\sin 8x + \sin 2x}{4x \cos 3x}$
227. $\lim_{x \rightarrow 0} \frac{\sin 2x}{3 - \sqrt{2x+9}}$
228. $\lim_{x \rightarrow 0} \left(\frac{\sin 5x - \sin 2x}{\sin 8x - \sin 3x} \right)$
229. $\lim_{x \rightarrow 0} \left(\frac{\tan 2x - \tan x}{\sin 2x - \sin x} \right)$
230. $\lim_{x \rightarrow \frac{p}{4}} \left(\frac{1 - \tan x}{\frac{p}{4} - x} \right)$
231. $\lim_{x \rightarrow \frac{p}{2}} \left(\frac{1 - \cos 4x}{x \sin x} \right)$
232. $\lim_{x \rightarrow \frac{p}{2}} \left(\frac{\sin(\cos x)}{\cos x} \right)$
233. $\lim_{x \rightarrow \frac{1}{4}p} \frac{\cos x - \sin x}{x - \frac{1}{4}p}$
234. $\lim_{x \rightarrow \frac{1}{2}p} \frac{\sin x - \cos x}{1 - \sin 2x}$
235. $\lim_{x \rightarrow 1} \frac{\sin(x^2 - 1)}{x - 1}$
236. $\lim_{x \rightarrow \frac{1}{2}p} \frac{1 + \cos 2x}{\cos x}$
237. $\lim_{x \rightarrow a} \frac{3(x-a)}{\sin(x-a) + 2x - 2a}$
238. $\lim_{x \rightarrow 1} \frac{x^3 - (a+1)x^2 + ax}{(x^2 - a) + \tan(x-1)}$
239. $\lim_{x \rightarrow p} \frac{1 + \cos x}{x - p}$
240. $\lim_{x \rightarrow 0} \frac{\sin 2x(1 + \cos x)}{\tan x(1 + 3 \sec x)}$

$$241. \lim_{x \rightarrow 0} \frac{3 \sin 2x - 2 \sin 3x}{x(1 - \cos 3x)}$$

$$242. \lim_{x \rightarrow 0} \frac{x^3}{\sin 2x - \tan 2x}$$

$$243. \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$244. \lim_{x \rightarrow -3} \frac{1 - \cos(x+3)}{x^2 + 6x + 9}$$

$$245. \lim_{x \rightarrow 0} \frac{\sin 2x + \sin 6x + \sin 10x - \sin 18x}{3 \sin x - \sin 3x}$$

$$246. \lim_{x \rightarrow y} \left(\frac{\tan x - \tan y}{1 - \frac{x}{y} + \left(1 - \frac{x}{y}\right) \tan x \tan y} \right) \quad **$$

E. Tentukan, jika ada, titik-titik yang menyebabkan fungsi-fungsi berikut tidak kontinu:

$$247. f(x) = \frac{x^2 - 1}{x^2 + x}$$

$$248. f(x) = \frac{x^2 + 2x + 3}{x^3 - 1}$$

$$249. f(x) = \frac{2x^2 - 5x - 3}{x^2 + x - 2}$$

$$250. f(x) = \frac{x^2 + 1}{x^2 + 3x - 10}$$

$$251. f(x) = \frac{2x + 1}{x^2 - x + 1}$$

$$252. f(x) = \begin{cases} 1; & \text{unt } x < 0 \\ 1 - x; & \text{unt } x \geq 0 \end{cases}$$

$$253. f(x) = \begin{cases} 2x; & \text{unt } x < 0 \\ -x; & \text{unt } x \geq 0 \end{cases}$$

$$254. f(x) = \begin{cases} x; & \text{unt } x < 0 \\ 1; & \text{unt } x = 0 \\ x^2; & \text{unt } x > 0 \end{cases}$$

$$255. f(x) = \frac{x^2 - 1}{x - 1}$$

$$256. f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}; & \text{unt } x \neq 1 \\ 2; & \text{unt } x = 1 \end{cases}$$

Selidikilah, apakah fungsi-fungsi berikut kontinu pada titik yang diberikan:

$$257. f(x) = 5, \text{ pada } x = 1$$

$$258. f(x) = 5x - 10, \text{ pada } x = -3$$

$$259. f(x) = \frac{8}{x - 3}, \text{ pada } x = 3$$

$$260. f(x) = \frac{3x - 12}{x^2 - 7x + 12}, \text{ pada } x = 4$$

$$261. f(x) = \frac{3x^2 + 3x - 6}{2x^2 - 2x - 12}, \text{ pada } x = -2$$

F. Hitunglah nilai dari limit fungsi berikut:

$$262. \lim_{x \rightarrow \infty} \left(\frac{x}{x+1} \right)^{x+1}$$

$$263. \lim_{x \rightarrow \infty} \left(1 + \frac{2}{3+x} \right)^{-2x}$$

$$264. \lim_{x \rightarrow \infty} \left(\frac{x+5}{x+3} \right)^{x+6}$$

$$265. \lim_{x \rightarrow \infty} \left(\frac{2x+2}{2x+6} \right)^{2x}$$

$$266. \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} \right)^x$$

$$267. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^{ax}$$

$$268. \lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x+5} \right)^{x^2+3}$$

$$269. \lim_{x \rightarrow \infty} \left(\frac{x^2 + 3x + 2}{x^2 + 5x + 1} \right)^{\frac{x^2+1}{x+1}}$$

$$270. \quad \lim_{x \rightarrow \infty} \left(\frac{x^2 + 3x + 2}{x^2 + 7x + 5} \right)^{\frac{5x^2 + 3x}{2x + 1}}$$

G. Hitunglah nilai dari $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
dari fungsi-fungsi berikut:

271. $f(x) = 9$

272. $f(x) = 5x$

273. $f(x) = 8x - 10$

274. $f(x) = x^2$

275. $f(x) = 3x^2$

276. $f(x) = -2x^2 + 1$

277. $f(x) = 2x^2 + 3x$

278. $f(x) = x^3$

279. $f(x) = 2x^3$

280. $f(x) = \sqrt{x}$

281. $f(x) = 2\sqrt{x}$

282. $f(x) = 2\sqrt{x+1}$

Kata-kata mutiara:

- Where there is a will, there is a way, Dimana ada kemauan, disitu pasti ada jalan.
- Practise makes perfect, banyak latihan kuncine kesuksesan.
- Witing tresno jalaran soko kulino, witing iso jalaran soko kerep nyobo.
- Kalau orang lain bisa, kita InsyaAlloh juga bisa.
- Gagal adalah kesuksesan yang tertunda, maju teruuuss...

Sumber:

- Matematika SMA XI, Erlangga, BK Noormandiri.
- Cerdas Belajar Matematika, Grafindo, Marthen Kanginan.
- Matematika SMA/MA XI, Gelora Aksara Pratama, Sulistiyono, dkk.
- Mathematics Year XI, Yudhistira, Team.
- Matematika unt SMA/MA XI, Piranti, Yanti M dkk.
- Matematika IPA kelas XI, Intan Pariwara, Kartini dkk.
- Matematika 2 SMU, Balai Pustaka, Andi Hakim N.
- Lainnya.