

## Jumlah Hasil Kali Akar-akar Polinom (2)

Misalkan Persamaan Polinom  $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x + a_0 = 0$  mempunyai akar-akar  $x_1, x_2, x_3, \dots, x_n$  maka jumlah hasil kali setiap  $k$  akar-akarnya adalah:

$$\frac{(-1)^k \cdot a_{n-k}}{a_n}; \quad k=1, 2, \dots, n$$

### Contoh:

Misalkan akar-akar dari  $a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0$  atau  $ax^3 + bx^2 + cx + d = 0$  adalah  $x_1, x_2$ , dan  $x_3$ . Maka:

- $x_1 + x_2 + x_3 = \frac{(-1)^1 a_{3-1}}{a_3} = \frac{-a_2}{a_3} = \frac{-b}{a}$
- $x_1 x_2 + x_1 x_3 + x_2 x_3 = \frac{(-1)^2 a_{3-2}}{a_3} = \frac{a_1}{a_3} = \frac{c}{a}$
- $x_1 x_2 x_3 = \frac{(-1)^3 a_{3-3}}{a_3} = \frac{-a_0}{a_3} = \frac{-d}{a}$

**Kemudian, kita dapat mencari bentuk-bentuk lainnya sebagai berikut:**

- $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = \frac{x_2 x_3}{x_1 x_2 x_3} + \frac{x_1 x_3}{x_1 x_2 x_3} + \frac{x_1 x_2}{x_1 x_2 x_3} = \frac{x_1 x_2 + x_1 x_3 + x_2 x_3}{x_1 x_2 x_3}$
- $\frac{1}{x_1 x_2} + \frac{1}{x_1 x_3} + \frac{1}{x_2 x_3} = \frac{x_3}{x_1 x_2 x_3} + \frac{x_2}{x_1 x_2 x_3} + \frac{x_1}{x_1 x_2 x_3} = \frac{x_1 + x_2 + x_3}{x_1 x_2 x_3}$
- $x_1^2 + x_2^2 + x_3^2 = (x_1 + x_2 + x_3)^2 - 2(x_1 x_2 + x_1 x_3 + x_2 x_3)$

Bukti:

$$\begin{aligned}(x_1 + x_2 + x_3)^2 &= (x_1 + x_2 + x_3)(x_1 + x_2 + x_3) \\ &= (x_1^2 + x_2^2 + x_3^2) + 2x_1 x_2 + 2x_1 x_3 + 2x_2 x_3 \\ &= (x_1^2 + x_2^2 + x_3^2) + 2(x_1 x_2 + x_1 x_3 + x_2 x_3) \\ (x_1^2 + x_2^2 + x_3^2) &= (x_1 + x_2 + x_3)^2 - 2(x_1 x_2 + x_1 x_3 + x_2 x_3)\end{aligned}$$

$$4. \frac{1}{x_1^2} + \frac{1}{x_2^2} + \frac{1}{x_3^2} = \frac{(x_1x_2 + x_1x_3 + x_2x_3)^2 - 2x_1x_2x_3(x_1 + x_2 + x_3)}{(x_1x_2x_3)^2}$$

Bukti:

$$\begin{aligned} \frac{1}{x_1^2} + \frac{1}{x_2^2} + \frac{1}{x_3^2} &= \frac{x_2^2x_3^2}{x_1^2x_2^2x_3^2} + \frac{x_1^2x_3^2}{x_1^2x_2^2x_3^2} + \frac{x_1^2x_2^2}{x_1^2x_2^2x_3^2} \\ &= \frac{x_1^2x_2^2 + x_1^2x_3^2 + x_2^2x_3^2}{(x_1x_2x_3)^2} \\ &= \frac{(x_1x_2)^2 + (x_1x_3)^2 + (x_2x_3)^2}{(x_1x_2x_3)^2} \\ &= \frac{(x_1x_2 + x_1x_3 + x_2x_3)^2 - 2(x_1^2x_2x_3 + x_1x_2^2x_3 + x_1x_2x_3^2)}{(x_1x_2x_3)^2} \\ &= \frac{(x_1x_2 + x_1x_3 + x_2x_3)^2 - 2x_1x_2x_3(x_1 + x_2 + x_3)}{(x_1x_2x_3)^2} \end{aligned}$$

$$\begin{aligned} 5. \frac{x_1}{x_2x_3} + \frac{x_2}{x_1x_3} + \frac{x_3}{x_1x_2} &= \frac{x_1^2 + x_2^2 + x_3^2}{x_1x_2x_3} \\ &= \frac{(x_1 + x_2 + x_3)^2 - 2(x_1x_2 + x_1x_3 + x_2x_3)}{x_1x_2x_3} \end{aligned}$$

Coba anda cari bentuk-bentuk lainnya, dari persamaan polinom berderajat 2, 3, atau mungkin 4.

$$6. \quad x_1^3 + x_2^3 + x_3^3 = (x_1 + x_2 + x_3)^3 + 3x_1x_2x_3 - 3(x_1 + x_2 + x_3)(x_1x_2 + x_1x_3 + x_2x_3)$$

Bukti:

$$\begin{aligned} (x_1 + x_2 + x_3)^3 &= (x_1 + x_2 + x_3)(x_1 + x_2 + x_3)(x_1 + x_2 + x_3) \\ &= (x_1^3 + x_2^3 + x_3^3) + 6x_1x_2x_3 + 3(x_1x_2^2 + x_1^2x_2 + x_1x_3^2 + x_1^2x_3 + x_2x_3^2 + x_2^2x_3) \end{aligned} \quad \dots\dots\dots 1$$

$$(x_1^2 + x_2^2 + x_3^2)(x_1 + x_2 + x_3) = (x_1^3 + x_2^3 + x_3^3) + (x_1x_2^2 + x_1^2x_2 + x_1x_3^2 + x_1^2x_3 + x_2x_3^2 + x_2^2x_3)$$

$$(x_1x_2^2 + x_1^2x_2 + x_1x_3^2 + x_1^2x_3 + x_2x_3^2 + x_2^2x_3) = (x_1^2 + x_2^2 + x_3^2)(x_1 + x_2 + x_3) - (x_1^3 + x_2^3 + x_3^3) \quad \dots\dots\dots 2$$

Substitusi 2 ke 1,

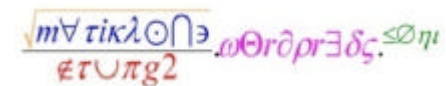
$$(x_1 + x_2 + x_3)^3 = (x_1^3 + x_2^3 + x_3^3) + 6x_1x_2x_3 + 3[(x_1^2 + x_2^2 + x_3^2)(x_1 + x_2 + x_3) - (x_1^3 + x_2^3 + x_3^3)]$$

karena  $(x_1^2 + x_2^2 + x_3^2) = (x_1 + x_2 + x_3)^2 - 2(x_1x_2 + x_1x_3 + x_2x_3)$ , maka

$$\begin{aligned} (x_1 + x_2 + x_3)^3 &= (x_1^3 + x_2^3 + x_3^3) + 6x_1x_2x_3 + 3[(x_1 + x_2 + x_3)^2 - 2(x_1x_2 + x_1x_3 + x_2x_3)](x_1 + x_2 + x_3) - 3(x_1^3 + x_2^3 + x_3^3) \\ &= (x_1^3 + x_2^3 + x_3^3) + 6x_1x_2x_3 + 3((x_1 + x_2 + x_3)^2 - 2(x_1x_2 + x_1x_3 + x_2x_3))(x_1 + x_2 + x_3) - 3(x_1^3 + x_2^3 + x_3^3) \\ &= -2(x_1^3 + x_2^3 + x_3^3) + 6x_1x_2x_3 + 3(x_1 + x_2 + x_3)^3 - 6(x_1x_2 + x_1x_3 + x_2x_3)(x_1 + x_2 + x_3) \end{aligned}$$

$$2(x_1^3 + x_2^3 + x_3^3) = 2(x_1 + x_2 + x_3)^3 + 6x_1x_2x_3 - 6(x_1x_2 + x_1x_3 + x_2x_3)(x_1 + x_2 + x_3)$$

$$(x_1^3 + x_2^3 + x_3^3) = (x_1 + x_2 + x_3)^3 + 3x_1x_2x_3 - (x_1x_2 + x_1x_3 + x_2x_3)(x_1 + x_2 + x_3)$$



dan lain sebagainya....