Rumus Sisa Pembagian Polinom dengan $(x - a)(x - b)(x - c)$

Cara Subtitusi

Suatu polinom (suku banyak) $f(x)$ jika dibagi oleh $p(x) = (x-a)(x-b)(x-c)$ mempunyai hasil bagi $h(x)$ dan sisa $s(x) = px^2 + qx + r$.
sehingga:

$$f(x) = p(x) \cdot h(x) + s(x)$$
$$= (x-a)(x-b)(x-c) \cdot h(x) + \left(px^2 + qx + r\right)$$

Untuk $x = a \quad \Rightarrow f(a) = a^2 p + aq + r$ .... (1)

$x = b \quad \Rightarrow f(b) = b^2 p + bq + r$ ....(2)

$x = c \quad \Rightarrow f(c) = c^2 p + cq + r$ .... (3)

persamaan (1) dikurangi persamaan (2)

$$f(a) - f(b) = \left(a^2 - b^2\right) p + (a-b) q \quad \Rightarrow \quad p = \frac{f(a) - f(b) - (a-b)q}{\left(a^2 - b^2\right)} \text{ ....(4)}$$

persamaan (2) dikurangi persamaan (3)

$$f(b) - f(c) = \left(b^2 - c^2\right) p + (b-c) q \quad \Rightarrow \quad p = \frac{f(b) - f(c) - (b-c)q}{\left(b^2 - c^2\right)} \text{ .... (5)}$$

dari persamaan (4) dan (5) diperoleh

$$\frac{f(a) - f(b) - (a-b)q}{\left(a^2 - b^2\right)} = \frac{f(b) - f(c) - (b-c)q}{\left(b^2 - c^2\right)}$$

$$\Rightarrow \frac{(b-c)q}{\left(b^2 - c^2\right)} - \frac{(a-b)q}{\left(a^2 - b^2\right)} = \frac{f(b) - f(a)}{\left(a^2 - b^2\right)} + \frac{f(b) - f(c)}{\left(b^2 - c^2\right)}$$

$$\Rightarrow \frac{q}{(b+c)} - \frac{q}{(a+b)} = \frac{f(b) - f(a)}{\left(a^2 - b^2\right)} + \frac{f(b) - f(c)}{\left(b^2 - c^2\right)}$$

$$\Rightarrow q\left(\frac{a-c}{(a+b)(b+c)}\right) = \frac{f(b) - f(a)}{\left(a^2 - b^2\right)} + \frac{f(b) - f(c)}{\left(b^2 - c^2\right)}$$

$$\Rightarrow q = \left(\frac{f(b) - f(a)}{\left(a^2 - b^2\right)} + \frac{f(b) - f(c)}{\left(b^2 - c^2\right)}\right)\frac{(a+b)(b+c)}{a-c}$$

$$\Rightarrow q = \left(\frac{(b+c)\left(f(b) - f(a)\right)}{(a-b)(a-c)} + \frac{(a+b)\left(f(b) - f(c)\right)}{(a-c)(b-c)}\right)$$

$$\Rightarrow q = \left(-\frac{(b+c)}{(a-b)(a-c)} f(a) - \frac{(a+c)}{(b-a)(b-c)} f(b) - \frac{(a+b)}{(c-a)(c-b)} f(c)\right)$$

subtitusi $q=\left(-\frac{(b+c)}{(a-b)(a-c)}f(a)-\frac{(a+c)}{(b-a)(b-c)}f(b)-\frac{(a+b)}{(c-a)(c-b)}f(c)\right)$

ke persamaan (4), diperoleh:

$$p=\frac{f(a)-f(b)-(a-b)q}{\left(a^2-b^2\right)}$$

$$=\frac{f(a)-f(b)-(a-b)\left(-\frac{(b+c)}{(a-b)(a-c)}f(a)-\frac{(a+c)}{(b-a)(b-c)}f(b)-\frac{(a+b)}{(c-a)(c-b)}f(c)\right)}{(a-b)(a+b)}$$

$$=\frac{f(a)-f(b)-\left(-\frac{(b+c)}{(a-c)}f(a)+\frac{(a+c)}{(b-c)}f(b)-\frac{(a-b)(a+b)}{(c-a)(c-b)}f(c)\right)}{(a-b)(a+b)}$$

$$=\frac{\left(\frac{(a-c)+(b+c)}{(a-c)}f(a)+\frac{-(b-c)-(a+c)}{(b-c)}f(b)+\frac{(a-b)(a+b)}{(c-a)(c-b)}f(c)\right)}{(a-b)(a+b)}$$

$$=\frac{\left(\frac{(a+b)}{(a-c)}f(a)+\frac{-(a+b)}{(b-c)}f(b)+\frac{(a-b)(a+b)}{(c-a)(c-b)}f(c)\right)}{(a-b)(a+b)}$$

$$=\frac{f(a)}{(a-b)(a-c)}+\frac{f(b)}{(b-a)(b-c)}+\frac{f(c)}{(c-a)(c-b)}$$

dari persamaan (1) $f(a)=a^2p+aq+r$, diperoleh

$$r=f(a)-a^2p-aq$$

$$=f(a)-a^2\left(\frac{f(a)}{(a-b)(a-c)}+\frac{f(b)}{(b-a)(b-c)}+\frac{f(c)}{(c-a)(c-b)}\right)$$
$$-a\left(-\frac{(b+c)}{(a-b)(a-c)}f(a)-\frac{(a+c)}{(b-a)(b-c)}f(b)-\frac{(a+b)}{(c-a)(c-b)}f(c)\right)$$

$$=\left(\frac{(a-b)(a-c)-a^2+a(b+c)}{(a-b)(a-c)}\right)f(a)+\frac{a(a+c)-a^2}{(b-a)(b-c)}f(b)+\frac{a(a+b)-a^2}{(c-a)(c-b)}f(c)$$

$$=\frac{bc}{(a-b)(a-c)}f(a)+\frac{ac}{(b-a)(b-c)}f(b)+\frac{ab}{(c-a)(c-b)}f(c)$$

subtitusi p, q, dan r ke $s(x) = px^2 + qx + r$, diperoleh:

$$
\begin{aligned}
s(x) &= px^2 + qx + r \\
&= \left(\frac{f(a)}{(a-b)(a-c)} + \frac{f(b)}{(b-a)(b-c)} + \frac{f(c)}{(c-a)(c-b)} \right) x^2 \\
&\quad + \left(-\frac{(b+c)\,f(a)}{(a-b)(a-c)} - \frac{(a+c)\,f(b)}{(b-a)(b-c)} - \frac{(a+b)\,f(c)}{(c-a)(c-b)} \right) x \\
&\quad + \left(\frac{bc.f(a)}{(a-b)(a-c)} + \frac{ac.f(b)}{(b-a)(b-c)} + \frac{ab.f(c)}{(c-a)(c-b)} \right) \\
&= \left(\frac{x^2-(b+c)x+bc}{(a-b)(a-c)} \right) f(a) + \left(\frac{x^2-(a+c)x+ac}{(b-a)(b-c)} \right) f(b) + \left(\frac{x^2-(a+b)x+ab}{(c-a)(c-b)} \right) f(c) \\
&= \frac{(x-b)(x-c)}{(a-b)(a-c)} f(a) + \frac{(x-a)(x-c)}{(b-a)(b-c)} f(b) + \frac{(x-a)(x-b)}{(c-a)(c-b)} f(c)
\end{aligned}
$$

Jadi, jika polinom / suku banyak $f(x)$ dibagi oleh $p(x) = (x-a)(x-b)(x-c)$ mempunyai sisa $s(x) = \frac{(x-b)(x-c)}{(a-b)(a-c)} f(a) + \frac{(x-a)(x-c)}{(b-a)(b-c)} f(b) + \frac{(x-a)(x-b)}{(c-a)(c-b)} f(c)$

Cara Horner

Misalkan polinom/suku banyak $f(x)$ dibagi oleh $p(x) = (x-a)(x-b)(x-c)$ dimana
$(x-a)$ sbg pembagi ke-1 atau $p_1(x)$ diperoleh $f(x) = (x-a)h_1(x) + s_1$
$(x-b)$ sbg pembagi ke-2 atau $p_2(x)$ diperoleh $h_1(x) = (x-b)h_2(x) + s_2$
$(x-c)$ sbg pembagi ke-3 atau $p_3(x)$ diperoleh $h_2(x) = (x-c)h_3(x) + s_3$, sehingga

$$
\begin{aligned}
f(x) &= (x-a)\,h_1(x) + s_1 \\
&= (x-a)\left[(x-b)h_2(x) + s_2\right] + s_1 \\
&= (x-a)\left[(x-b)\{(x-c)h_3(x) + s_3\} + s_2\right] + s_1 \\
&= (x-a)\left[\{(x-b)(x-c)h_3(x) + (x-b)s_3\} + s_2\right] + s_1 \\
&= (x-a)(x-b)(x-c)h_3(x) + (x-a)(x-b)s_3 + (x-a)s_2 + s_1
\end{aligned}
$$

$$
\begin{aligned}
s(x) &= (x-a)(x-b)s_3 + (x-a)s_2 + s_1 \\
&= p_1 \cdot p_2 \cdot s_3 + p_1 \cdot s_2 + s_1
\end{aligned}
$$

Catatan: Jika pembagi tidak dapat difaktorkan, gunakan pembagian dengan cara bersusun (*etung pestol*)