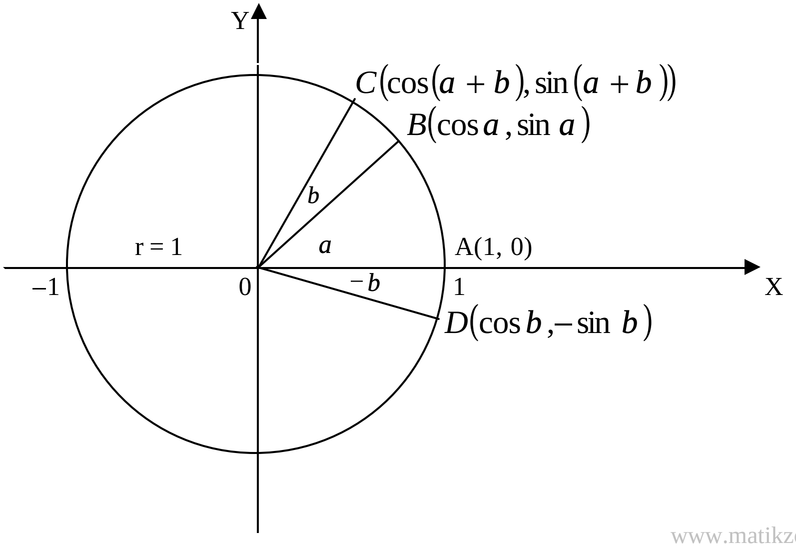
Rumus Cosinus, Sinus, dan Tangen untuk Jumlah/Selisih Dua Sudut



Berdasarkan rumus jarak: $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ dengan $P(x_1, y_1)$ dan $Q(x_2, y_2)$ maka:

$$\begin{aligned} AC^2 &= (\cos(a+b) - 1)^2 + (\sin(a+b) - 0)^2 \\ &= \cos^2(a+b) - 2\cos(a+b) + 1 + \sin^2(a+b) \\ &= \cos^2(a+b) + \sin^2(a+b) + 1 - 2\cos(a+b) \\ &= 1 + 1 - 2\cos(a+b) \\ &= 2 - 2\cos(a+b) \end{aligned}$$

$$
\begin{aligned}
BD^2 &= (\cos b - \cos a)^2 + (-\sin b - \sin a)^2 \\
&= \cos^2 b - 2\cos b \cos a + \cos^2 a + \sin^2 b + 2\sin b \sin a + \sin^2 a \\
&= (\cos^2 b + \sin^2 b) + (\cos^2 a + \sin^2 a) - 2\cos b \cos a + 2\sin b \sin a \\
&= 1 + 1 - 2\cos b \cos a + 2\sin b \sin a \\
&= 2 - 2\cos a \cos b + 2\sin a \sin b \\
&= 2 - 2(\cos a \cos b - \sin a \sin b)
\end{aligned}
$$

Oleh karena besar $\angle$BOD = $\angle$COA maka $AC^2 = BD^2$

$$2 - 2\cos(a + b) = 2 - 2(\cos a \cos b - \sin a \sin b)$$

$$\rightarrow \quad \cos(a + b) = \cos a \cos b - \sin a \sin b \; .$$

$$
\begin{aligned}
\cos(a - b) = \cos(a + (-b)) &= \cos a \cos(-b) - \sin a \sin(-b) \\
&= \cos a \cos b + \sin a \sin b
\end{aligned}
$$

$$\rightarrow \quad \cos(a - b) = \cos a \cos b + \sin a \sin b \; .$$

Berdasarkan pada $\cos\left(\tfrac{1}{2}p - l\right) = \sin l$ dan $\sin\left(\tfrac{1}{2}p - l\right) = \cos l$, maka:

$$
\begin{aligned}
\sin(a + b) &= \cos\left(\tfrac{1}{2}p - (a + b)\right) = \cos\left(\left(\tfrac{1}{2}p - a\right) - b\right) \\
&= \cos\left(\tfrac{1}{2}p - a\right)\cos b + \sin\left(\tfrac{1}{2}p - a\right)\sin b = \sin a \cos b + \cos a \sin b
\end{aligned}
$$

$$\rightarrow \quad \sin(a + b) = \sin a \cos b + \cos a \sin b \; .$$

$$\sin(a-b)=\sin(a+(-b))=\sin a\cos(-b)+\cos a\sin(-b)$$
$$=\sin a\cos b-\cos a\sin b$$

$$\Rightarrow\quad \sin(a-b)=\sin a\cos b-\cos a\sin b\ .$$

Dengan menggunakan $\tan x=\dfrac{\sin x}{\cos x}$, maka:

$$\tan(a+b)=\frac{\sin(a+b)}{\cos(a+b)}=\frac{\sin a\cos b+\cos a\sin b}{\cos a\cos b-\sin a\sin b}=\frac{\dfrac{\sin a\cos b}{\cos a\cos b}+\dfrac{\cos a\sin b}{\cos a\cos b}}{\dfrac{\cos a\cos b}{\cos a\cos b}-\dfrac{\sin a\sin b}{\cos a\cos b}}$$

$$=\frac{\dfrac{\sin a}{\cos a}+\dfrac{\sin b}{\cos b}}{1-\dfrac{\sin a}{\cos a}.\dfrac{\sin b}{\cos b}}=\frac{\tan a+\tan b}{1-\tan a\tan b}$$

$$\Rightarrow\quad \tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}\ .$$

$$\tan(a-b)=\tan(a+(-b))=\frac{\tan a+\tan(-b)}{1-\tan a\tan(-b)}=\frac{\tan a+(-\tan b)}{1-\tan a(-\tan b)}=\frac{\tan a-\tan b}{1+\tan a\tan b}$$

$$\Rightarrow\quad \tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}\ .$$