

Rumus Penjumlahan dan Perkalian Trigonometri

Bentuk Perkalian menjadi Penjumlahan/Pengurangan

$$\begin{array}{r} \cos(a+b) = \cos a \cos b - \sin a \sin b \\ \cos(a-b) = \cos a \cos b + \sin a \sin b \\ \hline \cos(a+b) + \cos(a-b) = 2 \cos a \cos b \end{array} +$$

$$\rightarrow \cos a \cos b = \frac{1}{2} \cos(a+b) + \frac{1}{2} \cos(a-b)$$

$$\begin{array}{r} \cos(a+b) = \cos a \cos b - \sin a \sin b \\ \cos(a-b) = \cos a \cos b + \sin a \sin b \\ \hline \cos(a+b) - \cos(a-b) = -2 \sin a \sin b \end{array} -$$

$$\rightarrow -\sin a \sin b = \frac{1}{2} \cos(a+b) - \frac{1}{2} \cos(a-b)$$

$$\begin{array}{r} \sin(a+b) = \sin a \cos b + \cos a \sin b \\ \sin(a-b) = \sin a \cos b - \cos a \sin b \\ \hline \sin(a+b) + \sin(a-b) = 2 \sin a \cos b \end{array} +$$

$$\rightarrow \sin a \cos b = \frac{1}{2} \sin(a+b) + \frac{1}{2} \sin(a-b)$$

$$\begin{array}{r} \sin(a+b) = \sin a \cos b + \cos a \sin b \\ \sin(a-b) = \sin a \cos b - \cos a \sin b \\ \hline \sin(a+b) - \sin(a-b) = 2 \cos a \sin b \end{array} -$$

$$\rightarrow \cos a \sin b = \frac{1}{2} \sin(a+b) - \frac{1}{2} \sin(a-b)$$

Bentuk Penjumlahan/Pengurangan menjadi Perkalian

Misalkan $a + b = P$ dan $a - b = Q$ maka

$$\begin{array}{l} a + b = P \\ a - b = Q \end{array} \quad + \quad \text{dan} \quad \begin{array}{l} a + b = P \\ a - b = Q \end{array} \quad -$$
$$\frac{2a = P + Q}{a = \frac{1}{2}(P + Q)} \quad \text{dan} \quad \frac{2b = P - Q}{b = \frac{1}{2}(P - Q)}$$

Substitusi ke persamaan di atas, diperoleh:

$$\begin{aligned} \cos a \cos b &= \frac{1}{2} \cos(a + b) + \frac{1}{2} \cos(a - b) \\ \Rightarrow \cos \frac{1}{2}(P + Q) \cos \frac{1}{2}(P - Q) &= \frac{1}{2} \cos P + \frac{1}{2} \cos Q \\ \Rightarrow 2 \cos \frac{1}{2}(P + Q) \cos \frac{1}{2}(P - Q) &= \cos P + \cos Q \end{aligned}$$

$$\rightarrow \cos P + \cos Q = 2 \cos \frac{1}{2}(P + Q) \cos \frac{1}{2}(P - Q)$$

$$\begin{aligned} -\sin a \sin b &= \frac{1}{2} \cos(a + b) - \frac{1}{2} \cos(a - b) \\ \Rightarrow -\sin \frac{1}{2}(P + Q) \sin \frac{1}{2}(P - Q) &= \frac{1}{2} \cos P - \frac{1}{2} \cos Q \\ \Rightarrow -2 \sin \frac{1}{2}(P + Q) \sin \frac{1}{2}(P - Q) &= \cos P - \cos Q \end{aligned}$$

$$\rightarrow \cos P - \cos Q = -2 \sin \frac{1}{2}(P + Q) \sin \frac{1}{2}(P - Q)$$

$$\begin{aligned} \sin a \cos b &= \frac{1}{2} \sin(a + b) + \frac{1}{2} \sin(a - b) \\ \Rightarrow \sin \frac{1}{2}(P + Q) \cos \frac{1}{2}(P - Q) &= \frac{1}{2} \sin P + \frac{1}{2} \sin Q \\ \Rightarrow 2 \sin \frac{1}{2}(P + Q) \cos \frac{1}{2}(P - Q) &= \sin P + \sin Q \end{aligned}$$

$$\rightarrow \sin P + \sin Q = 2 \sin \frac{1}{2}(P + Q) \cos \frac{1}{2}(P - Q)$$

$$\begin{aligned} \cos a \sin b &= \frac{1}{2} \sin(a + b) - \frac{1}{2} \sin(a - b) \\ \Rightarrow \cos \frac{1}{2}(P + Q) \sin \frac{1}{2}(P - Q) &= \frac{1}{2} \sin P - \frac{1}{2} \sin Q \\ \Rightarrow 2 \cos \frac{1}{2}(P + Q) \sin \frac{1}{2}(P - Q) &= \sin P - \sin Q \end{aligned}$$

$$\rightarrow \sin P - \sin Q = 2 \cos \frac{1}{2}(P + Q) \sin \frac{1}{2}(P - Q)$$