Rumus Turunan $f(x) = x^n$

Jika diketahui fungsi $f(x)$ maka turunan pertama dari $f(x)$ adalah $f'(x)$ dimana

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Soal : Tentukan turunan dari $f(x) = x^n$

Jawab :

$f(x) = x^n$

$f(x+h) = (x+h)^n$

Berdasarkan aturan binomial

$(a+b)^n = C(n,0)a^n + C(n,1)a^{n-1}b + ... + C(n,n-1)ab^{n-1} + C(n,n)b^n$ dan

Rumus kombinasi $C(n,r) = \dfrac{n!}{(n-r)!r!}$, diperoleh:

$f(x+h) = (x+h)^n = C(n,0)x^n + C(n,1)x^{n-1}h + ... + C(n,n-1)xh^{n-1} + C(n,n)h^n$

Oleh karena $C(n,0) = \dfrac{n!}{n!0!} = 1$ dan $C(n,n) = \dfrac{n!}{0!n!} = 1$, maka

$$\begin{aligned} f(x+h) - f(x) &= \left[x^n + C(n,1)x^{n-1}h + ... + C(n,n-1)xh^{n-1} + h^n\right] - x^n \\ &= C(n,1)x^{n-1}h + ... + C(n,n-1)xh^{n-1} + h^n \\ &= h\left[C(n,1)x^{n-1} + ... + C(n,n-1)xh^{n-2} + h^{n-1}\right] \end{aligned}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{h\left[C(n,1)x^{n-1} + ... + C(n,n-1)xh^{n-2} + h^{n-1}\right]}{h} \\ &= C(n,1)x^{n-1} + ... + C(n,n-1)xh^{n-2} + h^{n-1} \end{aligned}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} C(n,1)x^{n-1} + \left[C(n,2)x^{n-2}h ... + C(n,n-1)xh^{n-2} + h^{n-1}\right]$$

Semua suku yang berada di dalam kurung [ ] bernilai 0, karena semua mengandung variabel *h*

Sehingga $f'(x) = \lim_{h \to 0} C(n,1)x^{n-1} = C(n,1)x^{n-1} = \dfrac{n!}{(n-1)!1!}x^{n-1} = \dfrac{n(n-1)!}{(n-1)!}x^{n-1} = n.x^{n-1}$

$$f(x) = x^n \Rightarrow f'(x) = n.x^{n-1}$$

dan berlaku juga $f(x) = ax^n \Rightarrow f'(x) = an.x^{n-1}$, a : konstanta