Rumus Turunan Pipolondo

Diketahui fungsi $u(x)$ dan $v(x)$ adalah fungsi-fungsi yang mempunyai turunan.

Penjumlahan dan Pengurangan

$$f(x) = u(x) + v(x)$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[u(x+h)+v(x+h)]-[u(x)+v(x)]}{h}$$

$$= \lim_{h\to 0} \left[\frac{u(x+h)-u(x)}{h} + \frac{v(x+h)-v(x)}{h}\right]$$

$$= \lim_{h\to 0} \frac{u(x+h)-u(x)}{h} + \lim_{h\to 0} \frac{v(x+h)-v(x)}{h}$$

$$= u'(x) + v'(x)$$

Dengan cara yang sama, turunan $f(x) = u(x) - v(x)$ adalah $f'(x) = u'(x) - v'(x)$

➔ $f(x) = u(x) \pm v(x) \Rightarrow f'(x) = u'(x) \pm v'(x)$.

Atau $f(x) = u \pm v \Rightarrow f'(x) = u'v \pm uv'$.

Perkalian

$$f(x) = u(x).v(x)$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[u(x+h).v(x+h)]-[u(x).v(x)]}{h}$$

$$= \lim_{h\to 0} \frac{u(x+h).v(x+h) \underbrace{- u(x).v(x+h) + u(x).v(x+h)}_{=0} - u(x).v(x)}{h}$$

$$= \lim_{h\to 0} \left[v(x+h)\left(\frac{u(x+h)-u(x)}{h}\right) + u(x)\left(\frac{v(x+h)-v(x)}{h}\right)\right]$$

$$= u'(x)v(x) + u(x)v'(x)$$

➔ $f(x) = u(x).v(x) \Rightarrow f'(x) = u'(x)v(x) + u(x)v'(x)$.

Atau $f(x) = u.v \Rightarrow f'(x) = u'v + uv'$.

Pembagian

$$f(x) = \frac{u(x)}{v(x)}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{u(x+h)}{v(x+h)} - \frac{u(x)}{v(x)}}{h}$$

$$= \lim_{h\to 0} \frac{v(x).u(x+h) - u(x).v(x+h)}{h.v(x+h).v(x)}$$

$$= \lim_{h\to 0} \frac{u(x+h).v(x) - u(x).v(x+h) + \overbrace{u(x).v(x) - u(x).v(x)}^{=0}}{h.v(x+h).v(x)}$$

$$= \lim_{h\to 0} \frac{u(x+h).v(x) - u(x).v(x) - u(x).v(x+h) + u(x).v(x)}{h.v(x+h).v(x)}$$

$$= \lim_{h\to 0} \frac{\left[\frac{u(x+h) - u(x)}{h}\right].v(x) - \left[\frac{v(x+h) - v(x)}{h}\right].\, u(x)}{v(x+h).v(x)}$$

$$= \frac{u'(x).v(x) - v'(x).u(x)}{v(x).v(x)}$$

$$= \frac{u'(x).v(x) - v'(x).u(x)}{(v(x))^2}$$

$$f(x) = \frac{u(x)}{v(x)} \Rightarrow f'(x) = \frac{u'(x).v(x) - u(x).v'(x)}{(v(x))^2};\ v(x) \neq 0$$.

Atau $f(x) = \frac{u}{v} \Rightarrow f'(x) = \frac{u'.v - u.v'}{v^2};\ v \neq 0$.